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# The Magnetic Field Dynamics Equation of the Accreting and Rapidly Rotating Neutron Star in the ZAMO (Zero Angular Momentum Observers) Frame

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**Abstract.** The presence of accretion around neutron stars is assumed to be one of the causes of the decreasing of the magnetic field around neutron stars. Therefore, the equation that describes the relation between the decreasing of the magnetic fields and the accretion are required. The equation of the magnetic field decreasing describes the dynamics of the magnetic field around rapidly rotating neutron star. The relativistic magnetic field dynamics equation around a accreting and rapidly rotating neutron star in the ZAMO (Zero Angular Momentum Observers) frame has been formulated. The equation is derived from the rapidly rotating neutron star metric and the first and second of the relativistic Maxwell equations. The magnetic field dynamics equation is a differential equation in the radial, polar, and azimuthal parameter.

***Keyword**: Magnetic field, neutron stars, rapidly, ZAMO*

## 1. Introduction

Stars with mass $M_* \gtrsim 8M_\odot$, where $M_\odot$ denotes Solar mass, can undergo nucleosynthesis process or nuclear reactions. The nuclear reaction occurs in the form of burning silicon $^{28}Si$ to burning iron $^{56}Fe$. If the nucleosynthesis had finished, then the star collapsed and the supernova explosion occurred. After undergoing a supernova explosion, the state of the star depends on the thrown material. If a number of materials in the star's core after undergoing supernovae collapse, cool, reach an equilibrium and contain an abundance of degenerated neutrons, the star is called a neutron star [1]. In other words, neutron stars are produced by $M_* \gtrsim 8M_\odot$ massed massive stars that undergo supernova explosions [1]. The abundance of neutrons are obtained from the beta decay process, i.e., $+e^- \to n + v_e$ which occurs at high density.

A neutron star has a mass of $M_* \sim 1 - 2\, M_\odot$ and radius $R_* \approx 10 - 14$ km [2]. The maximum mass of neutron star is about $M_* \sim 1{,}5\, M_\odot$ with its radius is about $R_* \sim 3$ km [3]. The density of neutron star is about $\rho \simeq (2-3)\rho_o$, where $\rho_o = 2{,}8\, \times 10^{14}$ g cm$^{-3}$ is the normal density of the nucleus [4]. The gravitational potential of a neutron star is about $E_{\text{grav}} \sim GM^2/R \sim 5 \times 10^{53}\text{erg} \sim 0{,}2\, Mc^2$, whereas the gravitational field at a neutron star's surface is about $g \sim GM/R^2 \sim 2 \times 10^{14}\text{cm s}^{-2}$ [4].

The very large density and gravitational fields of neutron stars indicate that neutron stars are relativistic objects. Therefore, to understand all of the various physical phenomena both inside and outside of neutron star and around it, it can be explained by the general relativity approach. General relativistic aspects are determined by the compactness parameters as shown in equation (1.1)

$$x_g = r_g/R, \text{ with } r_g = 2GM/c^2 \approx 2{,}95\ M_*/M_\odot\ , \tag{1.1}$$

where $r_g$, $G$, and $c$ denote the radius of Schwarzschild, gravitational constant, and the speed of light, respectively. In general, stars have a compactness parameter is about $x_g \ll 1$, for example, $x_g \sim 10^{-4}$ for white dwarfs, and $x_g \sim 10^{-6}$ for the main row star with $M_* \simeq M_\odot$ [5]. By substituting the mass, $M_* \sim 1{,}4\ M_\odot$, and radius, $R_* \approx 12$ km, of the neutron star into equation (1.1), then we obtain $x_g = 0{,}3$.

The fact that neutron stars are rapidly rotating astrophysical objects also requires us to do a general relativistic approach to consider various physical phenomena in neutron stars. The rapidly rotating neutron star is one of the astrophysical objects that is classified into a gravitational field class. The very rapidly rotating neutron star is called the pulsar [6]. Numerically, the explanation of rapidly rotating neutron star is the solution of Einstein's field equations. There are two main influences that differentiate the rotating relativistic star from its non-rotating pair, i.e., the form of a star is flattened by centrifugal force. Moreover, the local inertial frame is "pulled" by rotation from the source of the gravitational field. To study the "pull" of inertial frame in the spacetime of rotating neutron star can be assisted by ZAMO (Zero Angular-Momentum Observers).

The existence of pulsar is known from electromagnetic waves. There are two types of electromagnetic waves, i.e., radio waves and X-rays. The emission will show the magnetic field possessed by the neutron star [7]. If the pulses are radio pulsars, the magnitude of the neutron star magnetic field is $B \sim 10^8\text{G} - 10^{13}\text{G}$. Whereas the magnetic field of X-ray pulses is about $B > 10^{12}\text{G}$ . The magnetic field is obtained from radio pulsars that emit infrared and accrete X-ray pulsars and LMXBs (Low Mass X-ray Binaries). The magnetic field inside the neutron star reaches of at least $\sim 10^{15}$G [8] or perhaps up to $10^{18}$ G, while the pulsar magnetic field is about $B \sim 10^{15}$G [9]. The neutron star magnetic field is the strongest magnetic field found in the universe [9].

Magnetic fields in neutron stars can decrease, for example, a neutron star that has a magnetic field of $\sim 10^{12}$ G becomes $\sim 10^8$ G in $5 \times 10^6$ years [10]. Neutron stars that experience a decrease in magnetic fields are found in multiple systems. A neutron star in a dual system shows a neutron star that performs the accretion process. Many astrophysicists therefore argue that the reduction of magnetic fields in neutron stars is due to the accreting neutron stars [11-17].

Cumming et al. have derived the equation that describes the decrease of the magnetic field in neutron stars due to accretion, for non-relativistic studies and non-rotating neutron star case. They obtained the equation of magnetic field dynamics related to the accretion velocity. Whereas the relativistic studies were introduced by Anderson et al. [18] that yielded the stationary electromagnetic fields in the Schwarzchild spacetime. The electromagnetic field is required to study the dynamics of magnetic field. The equation of the electromagnetic field can be obtained from Maxwell equations. Sengupta [19] stated that the obtained electric field of Schwarzschild metric in the neutron star was not the solution of Maxwell equations. Sengupta [20] has considered the velocity of Ohmic decay in the Schwarzschild spacetime. The Ohmic decay is assumed responsible for the cause of the magnetic field in the neutron stars. The studies related to the relativistic slowly rotating neutron stars were proposed by Muslinov and Tsygan [21]. While the study of the solution of Maxwell equations in spacetime for the slowly rotating neutron star with the absence of material around it has been conducted by Rezolla et al. [22]. In other words, they considered non-accretion neutron star. The considered case is slowly rotating neutron stars in the ZAMO (Zero Angular Momentum Observers) observers. These studies are required for describing the magnetic field dynamics for rotating neutron stars [22]. Previously, Atsnaita et al. [23] have also derived the equations of magnetic field dynamics for slowly rotating neutron star with the presence of material around it.

To describe the decrease of the magnetic field in neutron stars due to accretion, we need to derive the equations of magnetic field dynamics for rapidly rotating neutron stars. The equation is obtained from the formulation of Maxwell equations on neutron stars with the ZAMO frame.

## 2. Research methods

This research is an analytical theoretical-mathematical study. The research procedure to obtain the equation of magnetic field dynamics for the accreting rapidly rotating neutron stars in the ZAMO frame is shown in Fig. 2.1 below

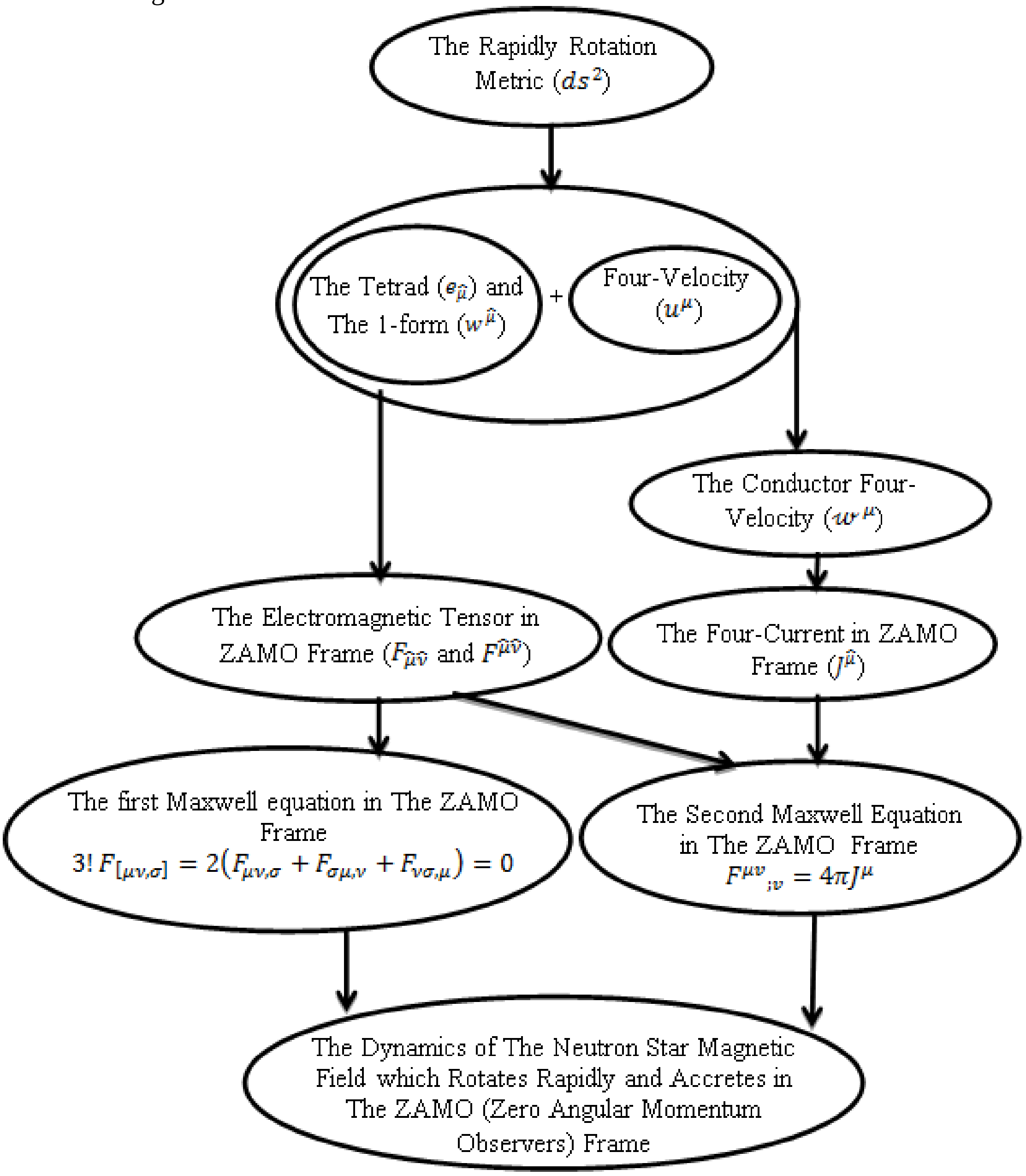


**Figure 2.1**. Research procedures to obtain the equation of magnetic field dynamics for the accreting rapidly rotating neutron stars in the ZAMO frame.

## 3. Results and discussion

*3.1. The rapidly rotating metric, tetrad, 1-form, and 4-velocity component for relativistic neutron stars*

The metric for rapidly rotating neutron star in the coordinate system $\{ t, r, \theta, \varphi \}$ is given by

$$ds^2 = -e^{2\phi}dt^2 + e^{2\lambda}r^2 \sin^2\theta\,(d\varphi - \omega dt)^2 + e^{2\alpha}(dr^2 + r^2 d\theta^2), \qquad (3.1)$$

where $\phi, \lambda, \omega, \alpha$ are the functions in the parameter of $r$ and $\theta$, respectively. The function of $\omega\,(r)$ is the angular velocity of the inertial reference frame [24]. The covariant metric component of rapidly rotating neutron stars is given by

$$g_{\mu\nu} = \begin{bmatrix} -(e^{2\phi} - e^{2\lambda}r^2\sin^2\theta\,\omega^2) & 0 & 0 & -e^{2\lambda}r^2\sin^2\theta\omega \\ 0 & e^{2\alpha} & 0 & 0 \\ 0 & 0 & e^{2\alpha}r^2 & 0 \\ -e^{2\lambda}r^2\sin^2\theta\,\omega & 0 & 0 & e^{2\lambda}r^2\sin^2\theta \end{bmatrix}. \qquad (3.2)$$

The tetrad $\{e_{\hat{\mu}}\} = (e_{\hat{0}}, e_{\hat{r}}, e_{\hat{\theta}}, e_{\hat{\varphi}})$ carried by ZAMO observer are

$$e_{\hat{0}}^{v} = e^{-\phi}(1,0,0,\omega), \qquad (3.3a)$$
$$e_{\hat{r}}^{v} = e^{-\alpha}(0,1,0,0), \qquad (3.3b)$$
$$e_{\hat{\theta}}^{v} = e^{-\alpha}r^{-1}(0,0,1,0), \qquad (3.3c)$$
$$e_{\hat{\varphi}}^{v} = \frac{e^{2\phi} - e^{2\lambda}r^2\sin^2\theta\omega^2}{e^{\phi}e^{\lambda}r\sin\theta}e^{-\phi}(0,0,0,1), \qquad (3.3d)$$

The 1-form $\{w^{\hat{\mu}}\} = (w^{\hat{0}}, w^{\hat{r}}, w^{\hat{\theta}}, w^{\hat{\varphi}})$, corresponding to this tetrad are given by [25]

$$w_{v}^{\hat{0}} = \left(e^{2\phi} - e^{2\lambda}r^2\sin^2\theta\,\omega^2\right)^{\frac{1}{2}}(1,0,0,0), \qquad (3.4a)$$
$$w_{v}^{\hat{r}} = e^{\alpha}(0,1,0,0), \qquad (3.4b)$$
$$w_{v}^{\hat{\theta}} = e^{\alpha}r(0,0,1,0), \qquad (3.4c)$$
$$w_{v}^{\hat{\varphi}} = e^{\lambda}r\sin\theta\,(-\omega,0,0,1). \qquad (3.4d)$$

The 4-velocity vector for the rapidly rotating neutron star metrics are [25]

$$u^{\mu} = \frac{e^{-\phi}}{\sqrt{1-V^2}}(1,0,0,\omega), \qquad (3.5a)$$
$$u_{\mu} = -\frac{e^{\phi}}{\sqrt{1-V^2}}(1,0,0,0), \qquad (3.5b)$$

where

$$V^2 = e^{2\phi}\left[e^{2\lambda}r^2\sin^2\theta\;(\Omega - \omega)^2 + e^{2\alpha}(v^r)^2 + e^{2\alpha}r^2\left(v^{\theta}\right)^2\right]. \qquad (3.6)$$

*3.2. The 4-velocity vector of conductor*

The 4 -velocity vector of a conductor can be obtained from the equation below

$$\vec{w} \equiv \delta\vec{u}, \qquad (3.7)$$

where $\delta$ denotes the existence of Eulerian perturbation [26]. The value of each component of velocity represents the perturbation is

$$\delta u^{\mu} = \Gamma\left(1, \delta v^{i}\right), \qquad (3.8)$$

where

$$\delta v^{i} = e_{i}^{v}\delta v^{i}. \qquad (3.9)$$

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By substituting Eqs. (3.3a) - (3.3d) and Eq. (3.9) into Eq. (3.15), then we obtain the 4-velocity vector of the conductor, i.e,

$$\delta u^{\mu} = \Gamma\left(1, e^{-\alpha}\delta v^{\hat{r}}, e^{-\alpha}r^{-1}\delta v^{\hat{\theta}}, \frac{(e^{2\Phi}-e^{2\lambda}r^2\omega^2\sin^2\theta)^{\frac{1}{2}}}{e^{\Phi+\lambda}\, r\sin\theta}\delta v^{\hat{\varphi}}\right) = w^{\alpha}, \tag{3.10}$$

where $\Gamma$ is [26]

$$\Gamma = \left[-g_{00}\left(1 + \frac{g_{\alpha\beta}(\delta v^{\alpha}\delta v^{\beta})}{g_{00}}\right)\right]^{-\frac{1}{2}}. \tag{3.11}$$

If the considered perturbation is weak, then $\delta v^{\alpha}\delta v^{\beta}$ for $\alpha = \beta$ is very small. The metric components in equation (3.2) are not zero if $\alpha = \beta$, then equation (3.18) becomes

$$\Gamma \approx [-g_{00}]^{-\frac{1}{2}} \approx \left(e^{2\Phi} - e^{2\lambda}r^2\omega^2\sin^2\theta\right)^{-\frac{1}{2}}. \tag{3.12}$$

If the neutron star is assumed as a conductor, then the relation $u^{\mu} = \Gamma(1,0,0,\Omega) = w^{\alpha}$ applies [26]. Therefore the 4-velocity vector of the conductor for the rapidly rotating neutron star is given by

$$\delta u^{\mu} = \Gamma\left(1,0,0, \frac{(e^{2\Phi}-e^{2\lambda}r^2\omega^2\sin^2\theta)^{\frac{1}{2}}}{e^{\Phi+\lambda}\, r\sin\theta}\delta v^{\hat{\varphi}}\right) = \Gamma(1,0,0,\Omega) = w^{\alpha}. \tag{3.13}$$

*3.3. The electromagnetic tensor in ZAMO frame*

The electromagnetic field tensor $F_{\alpha\beta}$ denotes the relation between the 4-vector of electric field $E^{\alpha}$ and magnetic field $B^{\alpha}$. The covariant component of the electromagnetic tensor component satisfies the equation

$$F_{\alpha\beta} \equiv 2u_{[\alpha}E_{\beta]} + \eta_{\alpha\beta\gamma\delta}u^{\gamma}B^{\delta}, \tag{3.14}$$

where $\eta_{\alpha\beta\gamma\delta}$ is pseudo-tensor. The pseudo-tensor expressed by the Levi-Civita symbol $\epsilon_{\alpha\beta\gamma\delta}$, are

$$\eta^{\alpha\beta\gamma\delta} = -\frac{1}{\sqrt{-g}}\epsilon_{\alpha\beta\gamma\delta}; \qquad \eta_{\alpha\beta\gamma\delta} = \sqrt{-g}\,\epsilon_{\alpha\beta\gamma\delta}, \tag{3.15}$$

Where

$$g \equiv \det\left|g_{\alpha\beta}\right| = -e^{2(\Phi+\Lambda)}r^4\sin^2\theta. \tag{3.16}$$

Therefore, the non-zero components of the covariant tensor of the electromagnetic field for the rapidly rotating neutron star are [25]

$$F_{00} = F_{11} = F_{22} = F_{33} = 0, \tag{3.17a}$$
$$F_{01} = -F_{10} = -YE_r - X\omega B^{\theta}, \tag{3.17b}$$
$$F_{02} = -F_{20} = -YE_{\theta} + X\omega B^{r}, \tag{3.17c}$$
$$F_{03} = -F_{30} = -YE_{\varphi}, \tag{3.17d}$$
$$F_{12} = -F_{21} = XB^{\varphi}, \tag{3.17e}$$
$$F_{13} = -F_{31} = -XB^{\theta}, \tag{3.17f}$$
$$F_{23} = -F_{32} = XB^{r}, \tag{3.17g}$$

where

$$X = \left(e^{2\alpha+\lambda}r^2\sin\theta\right)/\sqrt{1-V^2}, \tag{3.18}$$

and

$$Y = e^{\Phi}/\sqrt{1-V^2}. \tag{3.19}$$

Whereas the contravariant tensor components of the electromagnetic field are the inverse of the covariant tensor. The non-zero components of the contravariant tensor of the electromagnetic field for the rapidly rotating neutron star are

$$F^{00} = F^{11} = F^{22} = F^{33} = 0, \tag{3.20a}$$
$$F^{01} = -F^{10} = E^r/Y, \tag{3.20b}$$
$$F^{02} = -F^{20} = E^\theta/Y, \tag{3.20c}$$
$$F^{03} = -F^{30} = E^\phi/Y, \tag{3.20d}$$
$$F^{12} = -F^{21} = B_\varphi/X, \tag{3.20e}$$
$$F^{13} = -F^{31} = -(YB_\theta + X\omega E^r)/XY, \tag{3.20f}$$
$$F^{23} = -F^{32} = \left(YB_r - X\omega E^\theta\right)/XY. \tag{3.20g}$$

The covariant and contravariant tensor components of the electromagnetic field for the rapidly rotating neutron star are measured by ZAMO observer. They are obtained by changing the 4-vector of electric field $E^\alpha$ and magnetic field $B^\alpha$ in the ZAMO frame. The 4-vector of electric field $E^\alpha$ and magnetic field $B^\alpha$ in the ZAMO frame are given by

$$E^i = e^i_{\hat{\mu}} E^{\hat{\mu}}, \tag{3.21a}$$
$$E_i = g_{ii} e^i_{\hat{\mu}} E^{\hat{\mu}}, \tag{3.21b}$$
$$B^i = e^i_{\hat{\mu}} B^{\hat{\mu}}, \tag{3.21c}$$
$$B_i = g_{ii} e^i_{\hat{\mu}} B^{\hat{\mu}}. \tag{3.21d}$$

From the equation (3.22a) and (3.22b) then we get the 4-vector of electric field $E^\alpha$ for rapidly rotating neutron star in the ZAMO frame, i.e,

$$E_r = e^\alpha E^{\hat{r}}, \tag{3.22a}$$
$$E_\theta = e^\alpha r B^{\hat{\theta}}, \tag{3.22b}$$
$$E_\varphi = e^{\alpha-\phi} r \sin\theta \left(e^{2\phi} - e^{2\lambda} r^2 \omega^2 \sin^2\theta\right)^{\frac{1}{2}} E^{\hat{\varphi}}, \tag{3.22c}$$
$$E^r = e^{-\alpha} E^{\hat{r}}, \tag{3.22d}$$
$$E^\theta = e^{-\alpha} r^{-1} E^{\hat{\theta}}, \tag{3.22e}$$
$$E^\varphi = \frac{\left(e^{2\phi} - e^{2\lambda} r^2 \omega^2 \sin^2\theta\right)^{\frac{1}{2}}}{e^{\phi+\lambda}\, r \sin\theta} E^{\hat{\varphi}}. \tag{3.22f}$$

Whereas based on the equation (3.23c) and (3.23d) we can obtain the 4-vector of magnetic field $B^\alpha$ for rapidly rotating neutron star in the ZAMO frame as follow

$$B_r = e^\alpha B^{\hat{r}}, \tag{3.23a}$$
$$B_\theta = e^\alpha r B^{\hat{\theta}}, \tag{3.23b}$$
$$B_\varphi = e^{\alpha-\phi} r \sin\theta \left(e^{2\phi} - e^{2\lambda} r^2 \omega^2 \sin^2\theta\right)^{\frac{1}{2}} B^{\hat{\varphi}}. \tag{3.23c}$$
$$B^r = e^{-\alpha} B^{\hat{r}}, \tag{3.23d}$$
$$B^\theta = e^{-\alpha} r^{-1} B^{\hat{\theta}}, \tag{3.23e}$$
$$B^\varphi = \frac{\left(e^{2\phi} - e^{2\lambda} r^2 \omega^2 \sin^2\theta\right)^{\frac{1}{2}}}{e^{\phi+\lambda}\, r \sin\theta} B^{\hat{\varphi}}. \tag{3.23f}$$

By substituting equations (3.22a) - (3.22c) and (3.23d) - (3.23f) into equation (3.17a) - (3.17g), then we obtain the components of electromagnetic field covariant tensor for the rapidly rotating neutron stars measured in the ZAMO frame as follows

$$F_{\hat{0}\hat{0}} = F_{\hat{1}\hat{1}} = F_{\hat{2}\hat{2}} = F_{\hat{3}\hat{3}} = 0, \tag{3.24a}$$

$$F_{\hat{0}\hat{1}} = -F_{\hat{1}\hat{0}} = -\frac{e^{\phi+\alpha}}{\sqrt{1-V^2}} E^{\hat{r}} - \frac{\omega e^{\alpha+\lambda}}{\sqrt{1-V^2}} r \sin\theta\, B^{\hat{\theta}}, \quad (3.24b)$$
$$F_{\hat{0}\hat{2}} = -F_{\hat{2}\hat{0}} = -\frac{e^{\phi+\alpha}}{\sqrt{1-V^2}} r E^{\hat{\theta}} + \frac{\omega e^{\alpha+\lambda}}{\sqrt{1-V^2}} r^2 \sin\theta\, B^{\hat{r}}, \quad (3.24c)$$
$$F_{\hat{0}\hat{3}} = -F_{\hat{3}\hat{0}} = -\frac{e^{\lambda}}{\sqrt{1-V^2}} r \sin\theta \left(e^{2\phi} - e^{2\lambda} r^2 \omega^2 \sin^2\theta\right)^{\frac{1}{2}} E^{\hat{\varphi}}, \quad (3.24d)$$
$$F_{\hat{1}\hat{2}} = -F_{\hat{2}\hat{1}} = \frac{e^{-\phi+2\alpha}}{\sqrt{1-V^2}} r \left(e^{2\phi} - e^{2\lambda} r^2 \omega^2 \sin^2\theta\right)^{\frac{1}{2}} B^{\hat{\varphi}}, \quad (3.24e)$$
$$F_{\hat{1}\hat{3}} = -F_{\hat{3}\hat{1}} = -\frac{e^{\alpha+\lambda}}{\sqrt{1-V^2}} r \sin\theta\, B^{\hat{\theta}}, \quad (3.24f)$$
$$F_{\hat{2}\hat{3}} = -F_{\hat{3}\hat{2}} = \frac{e^{\alpha+\lambda}}{\sqrt{1-V^2}} r^2 \sin\theta\, B^{\hat{r}}. \quad (3.24g)$$

While the components of contravariant tensor of electromagnetic field for the rapidly rotating neutron stars measured in the ZAMO frame are obtained by substituting equations (3.22d) - (3.22f) and (3.23a) - (3.23c) into equation (3.20a) - (3.20f). The components of contravariant tensor of electromagnetic field for the rapidly rotating neutron stars measured in the ZAMO frame are

$$F^{\hat{0}\hat{0}} = F^{\hat{1}\hat{1}} = F^{\hat{2}\hat{2}} = F^{\hat{3}\hat{3}} = 0, \quad (3.25a)$$
$$F^{\hat{0}\hat{1}} = -F^{\hat{1}\hat{0}} = -e^{-\phi-\alpha} \sqrt{1-V^2} E^{\hat{r}}, \quad (3.25b)$$
$$F^{\hat{0}\hat{2}} = -F^{\hat{2}\hat{0}} = e^{-\phi-\alpha}\, r^{-1} \sqrt{1-V^2} E^{\hat{\theta}}, \quad (3.25c)$$
$$F^{\hat{0}\hat{3}} = -F^{\hat{3}\hat{0}} = \frac{\left(e^{2\phi} - e^{2\lambda} r^2 \sin^2\theta\, \omega^2\right)^{1/2}}{e^{2\phi+\lambda} r \sin\theta} \sqrt{1-V^2} B^{\hat{\varphi}}, \quad (3.25d)$$
$$F^{\hat{1}\hat{2}} = -F^{\hat{2}\hat{1}} = -\frac{\left(e^{2\phi} - e^{2\lambda} r^2 \sin^2\theta\, \omega^2\right)^{\frac{1}{2}}}{e^{\alpha+\phi+\lambda} r} \sqrt{1-V^2} B^{\hat{\varphi}}, \quad (3.25e)$$
$$F^{\hat{1}\hat{3}} = -F^{\hat{3}\hat{1}} = -\left[\frac{E^{\hat{\theta}}}{e^{\alpha+\lambda} r\, sin\theta} - \frac{\omega}{e^{\phi+\alpha}} B^{\hat{r}}\right] \sqrt{1-V^2}, \quad (3.25f)$$
$$F^{\hat{0}\hat{1}} = -F^{\hat{1}\hat{0}} = -\left[\frac{B^{\hat{\theta}}}{e^{\alpha+\lambda} r^2 \sin\theta} + \frac{\omega}{e^{\phi+\alpha}} E^{\hat{r}}\right]. \quad (3.25g)$$

*3.4. The 4 - current in ZAMO frame*

The conduction current $j^{\mu}$ carried by electrons with electrical conductivity $\sigma$ satisfies Ohm's law, that is

$$j_{\mu} = \sigma F_{\mu\nu} w^{\nu}. \quad (3.26)$$

Therefore, using equations (3.17a) - (3.17g), (3.13), and (3.26) the covariant components of the conduction current are

$$j_0 = -\sigma\Gamma\, Y E_{\varphi}, \quad (3.27a)$$
$$j_r = \sigma\Gamma \left(Y E_r - X B^{\theta}(\Omega - \omega)\right), \quad (3.27b)$$
$$j_{\theta} = \sigma\Gamma \left(Y E_{\theta} + X B^{r}(\Omega - \omega)\right), \quad (3.27c)$$
$$j_{\varphi} = \sigma\Gamma Y E_{\varphi}, \quad (3.27d)$$

While the contravariant components of the conduction current satisfies the equation

$$j^{\mu} = g^{\mu\nu} j_{\nu}. \quad (3.28)$$

Therefore the contravariant components of the conduction current are

$$j^{0} = e^{-2\phi} \sigma\Gamma\, Y E_{\varphi}(\Omega + \omega), \quad (3.29a)$$
$$j^{r} = e^{-2\alpha} \sigma\Gamma \left[Y E_r - X B^{\theta}(\Omega - \omega)\right], \quad (3.29b)$$
$$j^{\theta} = e^{-2\alpha} r^{-2} \sigma\Gamma \left[Y E_{\theta} + X B^{r}(\Omega - \omega)\right], \quad (3.29c)$$

$$j^{\varphi} = \sigma \Gamma \, Y E_{\varphi} [\frac{(e^{2\phi} + e^{2\lambda} r^2 \sin^2\theta \omega^2)}{e^{2\phi+2\lambda} r^2 \sin^2\theta} + e^{-2\phi} \omega \Omega]. \quad (3.29d)$$

The 4-current density satisfies the equation

$$J^{\mu} = \rho_e \mathcal{w}^{\nu} + j^{\mu}, \quad (3.30)$$

where $\rho_e$ is charge density, so the components of 4-current density are

$$J^0 = \Gamma\left[\rho_e + e^{-2\phi} \sigma Y \, E_{\varphi}(\Omega - \omega)\right], \quad (3.31a)$$
$$J^r = e^{-2\alpha} \, \sigma\Gamma\left[Y \, E_r - X B^{\theta}(\Omega - \omega)\right], \quad (3.31b)$$
$$J^{\theta} = e^{-2\alpha} \, r^{-2} \sigma\Gamma[Y \, E_{\theta} + X B^r(\Omega - \omega)], \quad (3.31c)$$
$$J^{\varphi} = \rho_e \Gamma\Omega + \sigma\Gamma Y \, E_{\varphi} \left[\frac{(e^{2\phi} + e^{2\lambda} r^2 \sin^2\theta \omega^2)}{e^{2\phi+2\lambda} r^2 \sin^2\theta} + e^{-2\phi} \omega\Omega\right]. \quad (3.31d)$$

The relation between the 4-current density $J^{\mu}$ and the 4-current density in the ZAMO frame $J^{\hat{\imath}}$ is

$$J^{\mu} = e^{\mu}_{\hat{\imath}} J^{\hat{\imath}}. \quad (3.32)$$

The 4-current density in the ZAMO framework for rapidly rotating neutron stars are given by

$$J^{\hat{0}} = e^{\phi} \rho_e \, \Gamma + \sigma \, e^{-2\phi+\lambda} r \sin\theta \, Y \, (\Omega - \omega) E^{\hat{\varphi}}, \quad (3.33a)$$
$$J^{\hat{r}} = \sigma\Gamma\left[Y E^{\hat{r}} - X \, e^{-2\alpha} r^{-1} B^{\hat{\theta}}(\Omega - \omega)\right], \quad (3.33b)$$
$$J^{\hat{\theta}} = \sigma\Gamma\left[Y E^{\hat{\theta}} + X \, e^{-2\alpha} r^{-1} B^{\hat{r}}(\Omega - \omega)\right], \quad (3.33c)$$
$$J^{\hat{\varphi}} = \Gamma\left[\rho_e(\Omega - \omega) \frac{(e^{\phi+\lambda} r \sin\theta)}{(e^{2\phi} - e^{2\lambda} r^2 \sin^2\theta \omega^2)^{\frac{1}{2}}} + \sigma \, Y r^2 \sin^2\theta \, E^{\hat{\varphi}}\right], \quad (3.33d)$$

*3.5. Maxwell equation in a rapidly rotating spacetime*

The general form of the first pair of general relativistic Maxwell equation is given by

$$3! \, F_{[\alpha\beta,\gamma]} = 2\left(F_{\alpha\beta,\gamma} + F_{\gamma\alpha,\beta} + F_{\beta\gamma,\alpha}\right) = 0. \quad (3.34)$$

By substituting equations (3.24a) - (3.24g) into equation (3.34) then we get relativistic Maxwell first equation for rapidly rotating neutron stars in the ZAMO frame, namely as

$$-\frac{e^{\phi+\alpha}}{\sqrt{1-V^2}} r\left(E^{\hat{\theta}}\right)_{,\varphi} + \omega \frac{e^{\alpha+\lambda}}{\sqrt{1-V^2}} r^2 \sin\theta \left(B^{\hat{r}}\right)_{,\varphi} + r\left(\frac{e^{\lambda}\sin\theta}{\sqrt{1-V^2}} \left(e^{2\phi} - e^{2\lambda} r^2 \sin^2\theta \, \omega^2\right)^{\frac{1}{2}} E^{\hat{\varphi}}\right)_{,\theta}$$
$$+ \frac{e^{\alpha+\lambda}}{\sqrt{1-V^2}} r^2 \sin\theta \frac{\partial B^{\hat{r}}}{\partial t} = 0, \quad (3.35a)$$
$$-\frac{e^{\phi+\alpha}}{\sqrt{1-V^2}} \left(E^{\hat{r}}\right)_{,\varphi} - \omega \frac{e^{\alpha+\lambda}}{\sqrt{1-V^2}} r^2 \sin\theta \left(B^{\hat{\theta}}\right)_{,\varphi} + \sin\theta \left(\frac{e^{\lambda} r}{\sqrt{1-V^2}} \left(e^{2\phi} - e^{2\lambda} r^2 \sin^2\theta \, \omega^2\right)^{\frac{1}{2}} E^{\hat{\varphi}}\right)_{,r}$$
$$-\frac{e^{\alpha+\lambda}}{\sqrt{1-V^2}} r \sin\theta \frac{\partial B^{\hat{\theta}}}{\partial t} = 0, \quad (3.35b)$$
$$-e^{\phi+\alpha}\left(\frac{E^{\hat{r}}}{\sqrt{1-V^2}}\right)_{,\varphi} - \omega \, r\left(\frac{e^{\alpha+\lambda}}{\sqrt{1-V^2}} \sin\theta \, B^{\hat{\theta}}\right)_{,\theta} + e^{\phi+\alpha}\left(\frac{r E^{\hat{\theta}}}{\sqrt{1-V^2}}\right)_{,r} - \sin\theta\left(\omega \frac{e^{\alpha+\lambda}}{\sqrt{1-V^2}} r^2 B^{\hat{r}}\right)_{,r}$$
$$+ \frac{e^{2\alpha-\phi}}{\sqrt{1-V^2}} r \left(e^{2\phi} - e^{2\lambda} r^2 \sin^2\theta \, \omega^2\right)^{\frac{1}{2}} \frac{\partial B^{\hat{\varphi}}}{\partial t} = 0. \quad (3.35c)$$

The relativistic Maxwell second equation satisfies the equation

$$F^{\mu\nu}{}_{;\nu} = 4\pi J^{\mu}. \tag{3.36}$$

Substituting equations (3.25a) - (3.25g) and (3.33a) - (3.33d) into equation (3.36), then we obtain the relativistic Maxwell second equation in the ZAMO frame that are

$$\left(e^{\alpha+\lambda} r^2 \sin\theta \sqrt{1-V^2} E^{\hat{r}}\right)_{,r} + \left(e^{\alpha+\lambda} r \sin\theta \sqrt{1-V^2} E^{\hat{\theta}}\right)_{,\theta} + \left(e^{-\phi+2\alpha} r \sqrt{1-V^2}\left(e^{2\phi} - e^{2\lambda} r^2 \sin^2\theta\, \omega^2\right)^{\frac{1}{2}} E^{\hat{\varphi}}\right)_{,\varphi} = 4\pi\Gamma\rho_e e^{2\phi+2\alpha+\lambda} r^2 \sin\theta + \frac{4\pi\Gamma e^{2\alpha+\lambda}}{\sqrt{1-V^2}} r^3 \sin^2\theta\,(\Omega-\omega) E^{\hat{\varphi}}, \tag{3.37a}$$

$$\left(-e^{\alpha+\lambda} r^2 \sin\theta \sqrt{1-V^2} \frac{\partial E^{\hat{r}}}{\partial t}\right)_{,r} + r\left(\sqrt{1-V^2} e^{\lambda} \sin\theta \left(e^{2\phi} - e^{2\lambda} r^2 \sin^2\theta\, \omega^2\right)^{1/2} B^{\hat{\varphi}}\right)_{,\theta} - e^{-\phi+\alpha} r \left(B^{\hat{\theta}}\right)_{,\varphi} - \omega e^{\alpha+\lambda} r^2 \sin\theta \left(E^{\hat{r}}\right)_{,\varphi} = \frac{4\pi\Gamma\rho_e e^{2\phi+2\alpha+\lambda}}{\sqrt{1-V^2}} r^2 \sin\theta\, E^{\hat{r}} - \frac{4\pi\Gamma e^{2\alpha+\lambda}}{\sqrt{1-V^2}} r^3 \sin^2\theta\,(\Omega-\omega) B^{\hat{\theta}}, \tag{3.37b}$$

$$\left(-e^{\alpha+\lambda} r \sin\theta \sqrt{1-V^2} \frac{\partial E^{\hat{\theta}}}{\partial t}\right)_{,r} - \sin\theta \left(\sqrt{1-V^2} e^{\alpha} r \left(e^{2\phi} - e^{2\lambda} r^2 \sin^2\theta\, \omega^2\right)^{\frac{1}{2}} B^{\hat{\varphi}}\right)_{,r} + e^{\phi+\alpha}\left(B^{\hat{r}}\right)_{,\varphi} - \omega e^{\alpha+\lambda} r \sin\theta \left(E^{\hat{\theta}}\right)_{,\varphi} = \frac{4\pi\Gamma\rho_e e^{2\phi+2\alpha+\lambda}}{\sqrt{1-V^2}} r^2 \sin\theta\, E^{\hat{\theta}} + \frac{4\pi\Gamma e^{\phi+2\alpha+\lambda}}{\sqrt{1-V^2}} r^3 \sin^2\theta\,(\Omega-\omega) B^{\hat{r}}, \tag{3.37c}$$

$$\left(-e^{-\phi+\alpha} r \left(e^{2\phi} - e^{2\lambda} r^2 \sin^2\theta\, \omega^2\right)^{\frac{1}{2}} \sqrt{1-V^2} \frac{\partial E^{\hat{\varphi}}}{\partial t}\right)_{,r} + \left(e^{\phi+\alpha} r B^{\hat{\theta}}\right)_{,r} - e^{\phi+\alpha}\left(B^{\hat{r}}\right)_{,\theta} - \left(\omega e^{\alpha+\lambda} r E^{\hat{r}}\right)_{,r} + r\,\omega\left(e^{\alpha+\lambda} E^{\hat{\theta}}\right)_{,\theta} = \frac{4\pi\Gamma\rho_e e^{2\phi+2\alpha+\lambda}}{\left(e^{2\phi} - e^{2\lambda} r^2 \sin^2\theta\, \omega^2\right)^{\frac{1}{2}}} r^3 \sin^2\theta\,(\Omega-\omega) + \frac{4\pi\sigma\Gamma e^{\phi+2\alpha+\lambda}}{\sqrt{1-V^2}} r^4 \sin^4\theta\, E^{\hat{\varphi}}, \tag{3.37d}$$

This equation corresponds to the slowly rotating neutron star case as conducted by Rezolla.

### *3.6. The dynamics equation of the magnetic field for the accreting and rapidly rotating neutron stars in the ZAMO frame*

As conducted by Rezolla [22], in this article we write the terms that contain $\omega$ and $\Omega$ as $\mathcal{O}(\Omega)$. Therefore the Maxwell second equation as shown by equations (3.37a) – (3.37d) can be written as follow

$$E^{\hat{r}} = \frac{\sqrt{1-V^2}}{4\pi\Gamma e^{2\phi+2\alpha+\lambda} r^2 \sin\theta}\left[e^{\alpha+\lambda} r^2 \sin\theta \sqrt{1-V^2} \frac{\partial E^{\hat{r}}}{\partial t} + r\left(\sqrt{1-V^2} e^{\lambda} \sin\theta \left(e^{2\phi} - e^{2\lambda} r^2 \sin^2\theta\, \omega^2\right)^{\frac{1}{2}} B^{\hat{\varphi}}\right)_{,\theta} - e^{-\phi+\alpha} r \left(B^{\hat{\theta}}\right)_{,\varphi}\right] + \mathcal{O}(\Omega). \tag{3.38a}$$

$$E^{\hat{\theta}} = \frac{\sqrt{1-V^2}}{4\pi\Gamma e^{2\phi+2\alpha+\lambda} r^2 \sin\theta}\left[\left(-e^{\alpha+\lambda} r \sin\theta \sqrt{1-V^2} \frac{\partial E^{\hat{\theta}}}{\partial t}\right)_{,r} - \sin\theta \left(\sqrt{1-V^2} e^{\alpha} r \left(e^{2\phi} - e^{2\lambda} r^2 \sin^2\theta\, \omega^2\right)^{\frac{1}{2}} B^{\hat{\varphi}}\right)_{,r} + e^{\phi+\alpha}\left(B^{\hat{r}}\right)_{,\varphi}\right] + \mathcal{O}(\Omega), \tag{3.38b}$$

$$E^{\hat{\varphi}} = \frac{\sqrt{1-V^2}}{4\pi\sigma\Gamma e^{\phi+2\alpha+\lambda}r^4\sin^4\theta}\left[-e^{-\phi+\alpha}\, r\sqrt{1-V^2}\left(e^{2\phi}-e^{2\lambda}r^2\sin^2\theta\,\omega^2\right)^{\frac{1}{2}}\frac{\partial E^{\hat{\varphi}}}{\partial t}+\left(e^{\phi+\alpha}\, rB^{\hat{\theta}}\right)_{,r}-\left(e^{\phi+\alpha}B^{\hat{r}}\right)_{,\theta}\right]+\mathcal{O}(\Omega). \quad (3.38c)$$

Furthermore, by substituting equations (3.38a) - (3.38c) into Maxwell first equation, then we obtained the dynamic equation of the magnetic field for the accreting and rapidly rotating neutron star in the ZAMO frame, that are

$$\frac{\partial B^{\hat{r}}}{\partial t} = \frac{\sqrt{1-V^2}}{(e^{\alpha+\lambda}r^2\sin\theta)}\left\{\frac{e^{-\phi-\alpha-\lambda}}{4\pi\Gamma r^2\sin\theta}\left[-e^{\alpha+\lambda}\, r^2\sin\theta\sqrt{1-V^2}\left(\frac{\partial E^{\hat{r}}}{\partial t}\right)_{,\varphi}-\sin\theta\left(\left(\sqrt{1-V^2}e^{\alpha}\, r\left(e^{2\phi}-e^{2\lambda}r^2\, sin^2\,\theta\,\omega^2\right)^{\frac{1}{2}}B^{\hat{\varphi}}\right)_{,r}\right)_{,\varphi}+e^{\phi+\alpha}\left(\left(B^{\hat{r}}\right)_{,\varphi}\right)_{,\varphi}\right]+\left(\mathcal{O}(\Omega)\right)_{,\varphi}-\omega\frac{e^{\alpha+\lambda}}{\sqrt{1-V^2}}r^2\sin\theta\left(B^{\hat{r}}\right)_{,\varphi}\right.$$
$$\left.-\left(\frac{r\sin\theta}{4\pi\sigma\Gamma e^{2\phi+2\alpha}}\left(e^{2\phi}-e^{2\lambda}r^2\sin^2\theta\,\omega^2\right)^{\frac{1}{2}}\left[-e^{-\phi+\alpha}\, r\sqrt{1-V^2}\left(e^{2\phi}-e^{2\lambda}r^2\sin^2\theta\,\omega^2\right)^{\frac{1}{2}}\frac{\partial E^{\hat{\varphi}}}{\partial t}+\left(e^{\phi+\alpha}\, rB^{\hat{\theta}}\right)_{,r}-\left(e^{\phi+\alpha}B^{\hat{r}}\right)_{,\theta}\right]+\mathcal{O}(\Omega)\right)_{,\theta}\right\}, \quad (3.39a)$$

$$\frac{\partial B^{\hat{\theta}}}{\partial t} = \frac{\sqrt{1-V^2}}{(e^{\alpha+\lambda}r\sin\theta)}\left\{-\frac{e^{-\phi-\alpha-\lambda}}{4\pi\Gamma r^2\sin\theta}\left[-e^{\alpha+\lambda}\, r^2\sin\theta\sqrt{1-V^2}\left(\frac{\partial E^{\hat{r}}}{\partial t}\right)_{,\varphi}+e^{\alpha}r\left(\left(\sin\theta\sqrt{1-V^2}\left(e^{2\phi}-e^{2\lambda}r^2\, sin^2\,\theta\,\omega^2\right)^{\frac{1}{2}}B^{\hat{\varphi}}\right)_{,\theta}\right)_{,\varphi}-e^{\phi+\alpha}\left(\left(B^{\hat{\varphi}}\right)_{,\theta}\right)_{,\varphi}\right]+\left(\mathcal{O}(\Omega)\right)_{,\varphi}-\omega\frac{e^{\alpha+\lambda}}{\sqrt{1-V^2}}r\sin\theta\left(B^{\hat{\theta}}\right)_{,\varphi}\right.$$
$$\left.+\sin\theta\left(\frac{r}{4\pi\sigma\Gamma e^{2\phi+2\alpha}}\left(e^{2\phi}-e^{2\lambda}r^2\sin^2\theta\,\omega^2\right)^{\frac{1}{2}}\left[-e^{-\phi+\alpha}\, r\sqrt{1-V^2}\left(e^{2\phi}-e^{2\lambda}r^2\sin^2\theta\,\omega^2\right)^{\frac{1}{2}}\frac{\partial E^{\hat{\varphi}}}{\partial t}+\left(e^{\phi+\alpha}\, rB^{\hat{\theta}}\right)_{,r}-\left(e^{\phi+\alpha}B^{\hat{r}}\right)_{,\theta}\right]+\mathcal{O}(\Omega)\right)_{,\theta}\right\}. \quad (3.39b)$$

$$\frac{\partial B^{\hat{\varphi}}}{\partial t} = \frac{\sqrt{1-V^2}}{(e^{-\phi+2\alpha}r(e^{2\phi}-e^{2\lambda}r^2\, sin^2\,\theta\,\omega^2)^{\frac{1}{2}}}$$
$$\left\{e^{\phi+\alpha}\left(\frac{1}{4\pi\Gamma r^2\sin\theta}\left[-e^{\alpha+\lambda}\, r^2\sin\theta\sqrt{1-V^2}\frac{\partial E^{\hat{r}}}{\partial t}+e^{\alpha}r\left(\sin\theta\sqrt{1-V^2}\left(e^{2\phi}-e^{2\lambda}r^2\, sin^2\,\theta\,\omega^2\right)^{\frac{1}{2}}B^{\hat{\varphi}}\right)_{,\theta}-e^{\phi+\alpha}\, r\left(B^{\hat{\theta}}\right)_{,\varphi}\right]+\mathcal{O}(\Omega)\right)_{,\theta}+\right.$$

$$\omega r\left(\frac{e^{\alpha+\lambda}}{\sqrt{1-V^2}}\sin\theta\, B^{\hat{\theta}}\right)_{,\theta} + e^{\phi+\alpha}\left(\frac{1}{4\pi\sigma\Gamma e^{2\phi+2\alpha} r\sin\theta}\left[-e^{\alpha+\lambda}\, r\sin\theta\sqrt{1-V^2}\,\frac{\partial E^{\hat{\theta}}}{\partial t} + e^{\phi+\alpha}\left(B^{\hat{r}}\right)_{,\varphi} - \right.\right.$$

$$\left.\left.\sin\theta\left(e^{\alpha} r\sqrt{1-V^2}\left(e^{2\phi} - e^{2\lambda} r^2 \sin^2\theta\,\omega^2\right)^{\frac{1}{2}} B^{\hat{\varphi}}\right)_{,r}\right] + \mathcal{O}(\Omega)\right)_{,r} + \sin\theta\left(\frac{\omega e^{\alpha+\lambda}}{\sqrt{1-V^2}} r^2\right)_{,r}\Bigg\}. \quad (3.39c)$$

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