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KATA PENGANTAR

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COMB INEQUALITIES FOR THE DEGREE CONSTRAINED MINIMUM SPANNING TREE PROBLEM

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ABSTRACT

The Degree Constrained Minimum Spanning Tree Problem (DCMST) is a problem of finding the minimum spanning tree in a given weighted graph (all weights are non negative), whilst also satisfies the degree requirement in every vertex. Since the DCMST can be formulated as MILP, then all constraints are valid inequalities, and among those constraints some are facets defining. In this paper we will discuss how to find constraints that constitute comb inequalities for the DCMST for vertex order 5 to 15 with increments 1.

Keywords: *degree constrained, minimum spanning tree, valid inequality, facet, comb*

1. INTRODUCTION

Even until very recently the Degree Constrained Minimum Spanning Tree Problem (DCMST) is not as popular as the Traveling Salesman Problem (TSP), but its application arises in many real life problems. The Degree Constrained Minimum Spanning Tree (DCMST) problem typically arises in the design of telecommunication, transportation and energy networks. It is concerned with finding a minimum-weight (distance or cost) spanning tree that satisfies specified degree restrictions on its vertices.

The DCMST may be used in the design of the road system, which has to serve a collection of suburbs/towns, and has the additional restriction that no more than certain number of roads (example: four roads) are allowed to meet at an intersection. The degree restrictions typically represent the capacity of a center (node) in the network. A degree constraint in a communication network also limits the liability in the case of vertex failure. In computer networks, the degree restrictions can be used to cater for the number of line interfaces available at a server/terminal. The problem is, apart from some trivial cases, computationally difficult (NP-complete)[1)].

To tackle the DCMST problem various methods have been developed, both exacts and heuristics. Until very recently exact algorithms have been restricted to solve only small sized problems. Thus, much of the literature work has focused on heuristics for example: variations of Prim's and Kruskal's algorithms in[2)] Genetics algorithm in[3)] Simulated Annealing in [4)], Iterative Refinement in [5] and [6], Tabu Search in [7-12)] and Modified Penalty[13)].

Cutting Plane as one of exact methods had been used to solve the DCMST, and the generated valid inequality defining facet or cut usually found by Gomory' s method According to Nemhauser and Wolsey[14)] a polyhedron $P \subseteq \mathbb{R}^n$ is defined as P = $\{x \in \mathbb{R}^n : Ax \le b\}$, where (A,b) is a m x (n+1) matrix. A bounded polyhedron is called as a polytope. An inequality $\pi x \le \pi_0$ is called as a valid inequality for P if it is satisfied by all points in P. If $\pi x \le \pi_0$ is a valid inequality for P and $F = \{x \in P: \pi x = \pi_0\}$, then F is called as a face for P and $\pi x \le \pi_0$ represents F. A face of P is proper if $F \neq \varnothing$ and $F \neq P$.

Given a graph G = (V,E), a cycle of G that does not contain all vertices is called a subtour. In a cycle every vertex has degree 2. Hence, if every edge in $G' \subseteq G$ satisfies $\sum_{i,j} x_{ij} = 2$, then such sub graph is called 2- matching.

The DCMST can be formulated as Mixed Integer Linear Programming as follow:

$$Minimise \quad \sum_{i}^{n}\sum_{j}^{n} c_{ij}x_{ij} \qquad (1)$$

Subject to:

$$\sum_{i,j} x_{ij} = n - 1 \qquad (2)$$

$$\sum_{i,j \in V'} x_{ij} \leq |V'| - 1, \qquad \forall \varnothing \neq V' \subseteq V \qquad (3)$$

$$1 \leq \sum_{j=1, i \neq j} x_{ij} \leq b_i \qquad i = 1,2,\ldots,n \qquad (4)$$

x_{ij} = 0 or 1 , $\quad 1 \leq i \neq j \leq n$. (5)

c_{ij} is the weight (or distance or cost) of the edge (i,j), b_i is the degree bound on vertex i and n is the number of vertices. Constraint (2) ensures that (n-1) edges are selected. Constraint (3) is the usual sub tour elimination constraints. Constraint (4) specifies the degree restriction on the vertices. The last constraint (5), is just the variable constraint, which restricts the variables to the value of 0 or 1. x_{ij} is 1 if the edge x_{ij} is selected or included in the tree T and 0, otherwise. This formulation is the most common formulation for the DCMST problem. However, due to some computational advantages, Calccetta and Hill[15)] replaced equations (2) and (4) with:

$$\sum_{j \in V} x_{ij} - d_i = 0, \qquad 1 \leq i \leq n \qquad (6)$$

$$\sum_{i=1}^{n} d_i = 2(n-1) \qquad (7)$$

$$1 \leq d_i \leq b_i \,, \qquad 1 \leq i \leq n \qquad (8)$$

This formulation has n additional variables (d's) but n fewer constraints. It was noted in [15] that this formulation was better computationally in a branch and cut procedure.

Comb is a valid inequality that consists of handle and teeth[16)]. A comb is a sub graph generated by a vertex set { H, T_1, T_2 , T_3,…, k} with the following properties :
$|H \cap T_i| \geq 1$, $\forall$ i=1,2,…,k.
$|T_i \setminus H| \geq 1$, $\forall$ i=1,2,…,k.
$2 \leq |T_i| \leq m - 2$, $\forall$ i=1,2,…,k, where m is the vertex order of the graph..
$T_i \cap T_j = \varnothing$, $i \neq j$
k is odd and at least 3.
In this paper we will discuss on other form of valid inequality for the DCMST problem which is comb inequality; and this paper is organized as follow: Section 1 gives the introduction; Section 2 briefly discuss the research 's methodology, and Section 3 discuss the observation' results about the comb inequalities, and finally the conclusion is given in Section 4.

2. MATERIALS AND METHODS

In this research we do the following steps to get the combs inequalities :

a. Collecting and gathering the relevant literature. This step is important because we must know if there is someone already done the same research to avoid duplication.

b. Generate data which form complete graph. Graphs generated are graphs with vertex order from 5 to 15 with increments 1.

c. Formulate those graphs as *Mixed Integer Linear Programming* (MILP) problems.

d. Observe the *handle, teeth* dan *comb* for those graphs.

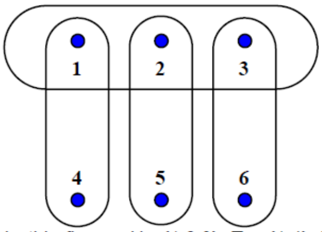
e. Determine if the *handle, teeth* dan *comb* make patterns.

3. RESULTS AND DISCUSSION

In this section we will discuss the comb inequalities for the DCMST where the vertex order maximum is 15. **Observation :**

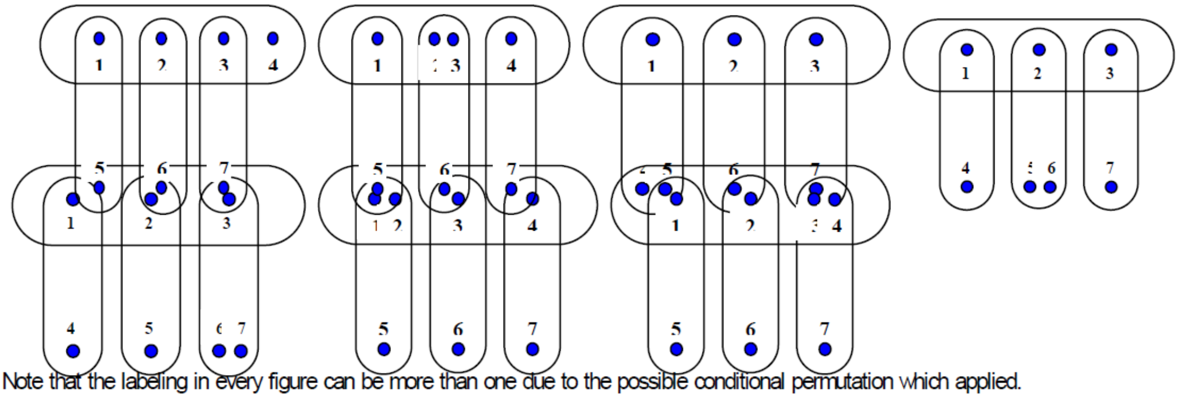
a. For vertex order maximum 5: there is no comb inequalities since there is no evidence the comb properties will be hold.

b. For vertex order 6 : The comb inequalities is a 2-matching inequalities and can be illustrated as follow:



In this figures H ={1,2,3}, T_1 ={1,4}, T_2={2,5}, T_3 ={3,6}. Note that this figure is one possibility among 20 possible conditional permutations that constitute comb where comb of that figure (and also 20 combs). The comb inequalities for this figures is $x_{12} + x_{13} + x_{23} + x_{14} + x_{25} + x_{36} \le 4$

c. For vertex order 7: the comb inequalities can take any figure from the figures below:



Note that the labeling in every figure can be more than one due to the possible conditional permutation which applied. Therefore, for figure on the top left hand side of vertex 7 for example, vertex 4 which is on the handle but not on the teeth can be put on the left hand side, in the middle(in the teeth and handle) or just on the teeth. Any isomorphic graph occurs is neglected. That conditional permutation applies for all figures. Due to space limitation, we do not give observation in picture for graphs with higher vertex order.

Next, we give some comb inequalities derived for graph with vertex order 15 (again, space limitation restricts us to give all results):

1.H = {1, 2, 3, 4, 5, 6, 7, 8}; T_1 = {1, 9}; T_2 = {2, 10}; T_3 = {3, 11}: T_4 = {4, 12}; T_5 = {5,13}; T_6 = {6, 14}: T_7 = {7, 15}

$$\sum_{e\in E(H)} x_e + \sum_{i=1}^{7} \sum_{e\in E(T_i)} x_e \le 11$$

2.H = {1, 2, 3, 4, 5, 6, 7}; T_1 = {1, 8, 9}; T_2 = {2, 10}; T_3 = {3, 11}: T_4 = {4, 12}; T_5 = {5,13}; T_6 = {6, 14}: T_7 = {7, 15}

$$\sum_{e\in E(H)} x_e + \sum_{i=1}^{7} \sum_{e\in E(T_i)} x_e \le 11$$

3.H = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}; T_1 = {1, 11}; T_2 = {2, 12}; T_3 = {3, 13}: T_4 = {4, 14}; T_5 = {5,15}

$$\sum_{e\in E(H)} x_e + \sum_{i=1}^{5} \sum_{e\in E(T_i)} x_e \le 12$$

4.H = {1, 2, 3, 4, 5, 6, 7, 8, 9}; T_1 = {1, 10, 11}; T_2 = {2, 12}; T_3 = {3, 13}: T_4 = {4, 14}; T_5 = {5,15}

$$\sum_{e\in E(H)} x_e + \sum_{i=1}^{5} \sum_{e\in E(T_i)} x_e \le 12$$

5.H = {1, 2, 3, 4, 5, 6, 7, 8}; T_1 = {1, 9, 10}; T_2 = {2, 11, 12}; T_3 = {3, 13}: T_4 = {4, 14}; T_5 = {5,15}

$$\sum_{e\in E(H)} x_e + \sum_{i=1}^{5} \sum_{e\in E(T_i)} x_e \le 12$$

6.H = {1, 2, 3, 4, 5, 6, 7}; T_1 = {1, 8, 9}; T_2 = {2, 10, 11}; T_3 = {3, 12, 13}: T_4 = {4, 14}; T_5 = {5,15}

$$\sum_{e\in E(H)} x_e + \sum_{i=1}^{5}\sum_{e\in E(T_i)} x_e \le 12$$

7.H = {1, 2, 3, 4, 5, 6}; T_1 = {1, 7, 8}; T_2 = {2, 9, 10}; T_3 = {3, 11, 12}: T_4 = {4, 13, 14}; T_5 = {5,15}

$$\sum_{e\in E(H)} x_e + \sum_{i=1}^{5}\sum_{e\in E(T_i)} x_e \le 12$$

8.H = {1, 2, 3, 4, 5}; T_1 = {1, 6, 7}; T_2 = {2, 8, 9}; T_3 = {3, 10, 11}: T_4 = {4, 12, 13}; T_5 = {5, 14, 15}

$$\sum_{e\in E(H)} x_e + \sum_{i=1}^{5}\sum_{e\in E(T_i)} x_e \le 12$$

9. H = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}; T_1 = {1, 13}; T_2 = {2, 14}; T_3 = {3, 15} $\sum_{e\in E(H)} x_e + \sum_{i=1}^{3}\sum_{e\in E(T_i)} x_e \le 13$

10. H = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}; T_1 = {1, 12, 13}; T_2 = {2, 14}; T_3 = {3, 15} $\sum_{e\in E(H)} x_e + \sum_{i=1}^{3}\sum_{e\in E(T_i)} x_e \le 13$

11. H = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}; T_1 = {1, 11, 12}; T_2 = {2, 13, 14}; T_3 = {3, 15} $\sum_{e\in E(H)} x_e + \sum_{i=1}^{3}\sum_{e\in E(T_i)} x_e \le 13$

12. H = {1, 2, 3, 4, 5, 6, 7, 8, 9}; T_1 = {1, 10, 11}; T_2 = {2, 12, 13}; T_3 = {3, 14, 15} $\sum_{e\in E(H)} x_e + \sum_{i=1}^{3}\sum_{e\in E(T_i)} x_e \le 13$

13. H = {1, 2, 3, 4, 5, 6, 7, 8}; T_1 = {1, 9, 10, 11}; T_2 = {2, 12, 13}; T_3 = {3, 14, 15} $\sum_{e\in E(H)} x_e + \sum_{i=1}^{3}\sum_{e\in E(T_i)} x_e \le 13$

14. H = {1, 2, 3, 4, 5, 6, 7}; T_1 = {1, 8, 9, 10}; T_2 = {2, 11, 12, 13}; T_3 = {3, 14, 15} $\sum_{e\in E(H)} x_e + \sum_{i=1}^{3}\sum_{e\in E(T_i)} x_e \le 13$

15. H = {1, 2, 3, 4, 5, 6}; T_1 = {1, 7, 8, 9}; T_2 = {2, 10, 11, 12}; T_3 = {3, 13, 14, 15} $\sum_{e\in E(H)} x_e + \sum_{i=1}^{3}\sum_{e\in E(T_i)} x_e \le 13$

16. H = {1, 2, 3, 4, 5}; T_1 = {1, 6, 7, 8}; T_2 = {2, 9, 10, 11}; T_3 = {3, 12, 13, 14, 15} $\sum_{e\in E(H)} x_e + \sum_{i=1}^{3}\sum_{e\in E(T_i)} x_e \le 13$

17. H = {1, 2, 3, 4}; T_1 = {1, 5, 6, 7, 8}; T_2 = {2, 9, 10, 11, 12}; T_3 = {3, 13, 14, 15} $\sum_{e\in E(H)} x_e + \sum_{i=1}^{3}\sum_{e\in E(T_i)} x_e \le 13$

18. H = {1, 2, 3}; T_1 = {1, 4, 5, 6, 7}; T_2 = {2, 8, 9, 10, 11}; T_3 = {3, 12, 13, 14, 15} $\sum_{e\in E(H)} x_e + \sum_{i=1}^{3}\sum_{e\in E(T_i)} x_e \le 13$

19. H_1 = {1, 2, 3}; H_2 = {8, 9, 10}; T_1 = {1, 4, 5}; T_2 = {2, 6, 7}; T_3 = {10, 13, 14, 15}; T_4 = {9, 11, 12}; T_5 = {3, 8}

$$\sum_{i=1}^{2}\sum_{e\in E(H)} x_e + \sum_{i=1}^{5}\sum_{e\in E(T_i)} x_e \le 12$$

4. CONCLUSION

Based on our observation we can conclude that there is no comb inequalities for vertex order lower than or equal to 5. For vertex order 6, comb inequality is also 2-matching. Graphs with vertex order multiplication of 3 and higher than 6 (such as 9, 12, and 15), also constitutes some 2-mathing among the combs inequalities for those graphs. Cliques occurs when the vertex order higher than or equal to 10.

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