ADVANCE MATHEMATICAL THINKING ABILITY AND ITS IMPACT ON PROBLEM SOLVING ABILITY

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Background

- In learning mathematics, educators must provide opportunities for students to see and think about the ideas that are given. For this reason, problem-solving is very important in learning

- Mathematics in higher education has shifted from elementary thinking to the formal framework of axiomatic systems and mathematical evidence

- Geometry as one of the subjects that is very dependent on the ability of advanced mathematical thinking. This study examines this ability and its influence on problem solving abilities.

- Mathematical thinking can be classified into two levels, namely low-level and high-level mathematical thinking ability
- The classification of low-level mathematical thinking includes working on simple arithmetic operations, using direct rules, doing algorithmic tasks
- The classification of high-level mathematical thinking includes meaningful understanding, constructing conjectures, making analogies and generalizations, logical reasoning, problem solving, and mathematical communication and connections
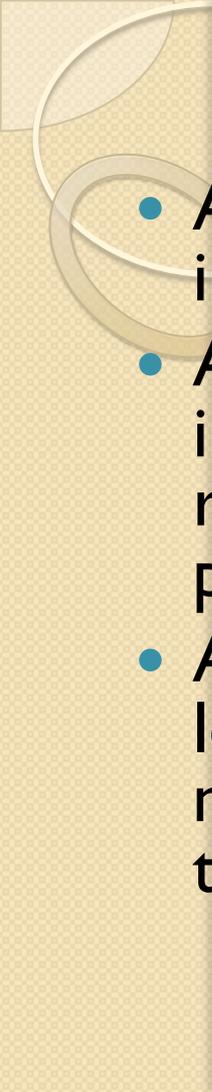
- Advanced mathematical thinking is concerned with the introduction of formal definitions and logical deduction
- Advanced Mathematics thinking is the ability which includes representation, abstraction, connecting representation and abstraction, and mathematical proofing
- Advanced mathematical thinking must contain high-level mathematical thinking. But not all high-level mathematical thinking includes advanced mathematical thinking.

- From many courses in university, geometry is one of the courses that require advanced thinking.

As an illustration, the following is presented an example problem in learning geometry.

On the ABC triangle, P is located in the middle of AB. Q lies in AC, so AQ: QC = 2: 1. Prove that BQ divides the CP segment into two equal parts!
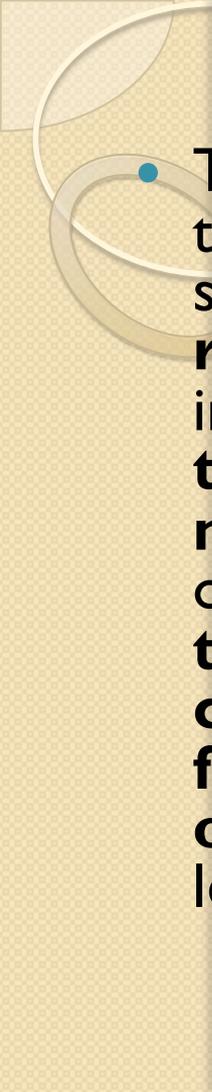
- To solve this problem, students are required to have the ability to **write a logical framework** to prove. For this reason, students must **present data and information from a representation in the form of graphical representations**, integrate **problem formulations and problem transformation into symbols, create mathematical models, and write mathematical formulas**. In the process of proof, students are required to be able to **generalize, link the formation of mathematical concepts with other concepts, associate further mathematical object formations with the formalization of mathematical objects, manipulate mathematical formulas** so that they lead to the correct conclusions, and then conclude.
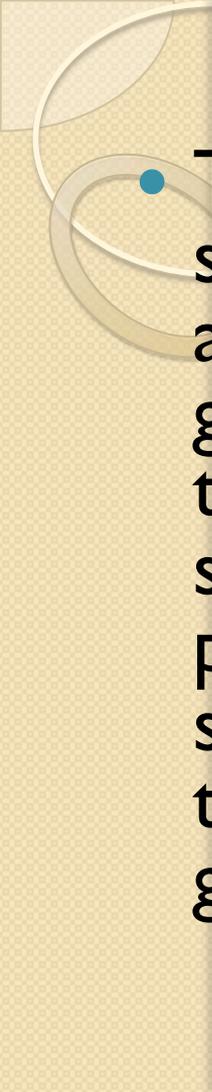
- This research discuss about how to utilize students in solving mathematical problems, what are the main problems for students in solving geometry task, which domains of advanced thinking ability are problems for students in solving geometry problems, and what kind of pedagogical practices are effective for developing students' abilities to solve geometry problems, in terms of advanced thinking skills and students' gender differences
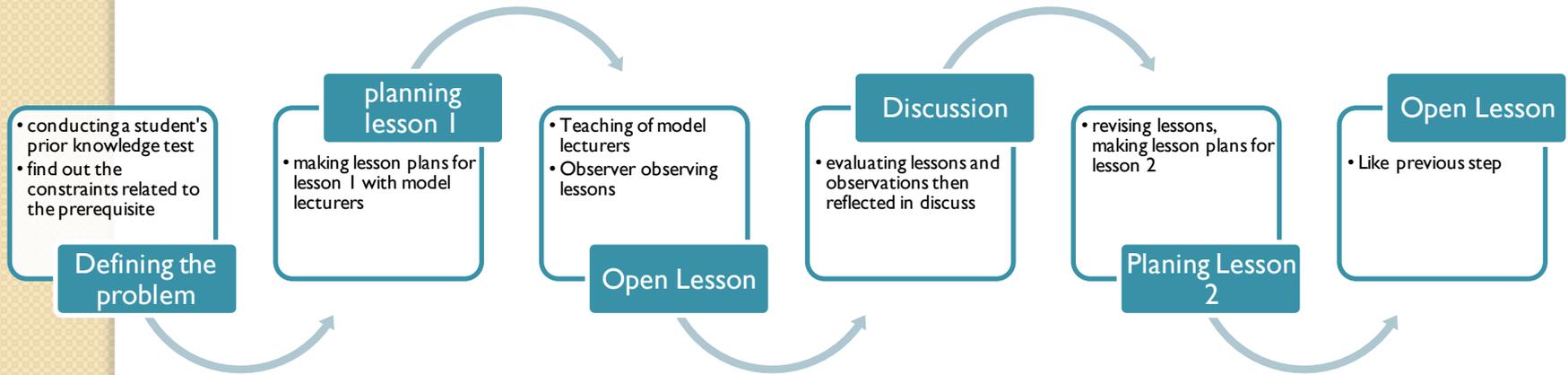
Methodology

- This research was conducted on students taking geometry courses in MESP, Department of MSE, Faculty of Teacher Training and Education University of Lampung, Indonesia

- The number of students are 65 students (55 female and 10 male students). The grouping is done by dividing students into 11 groups with distribution: 1 student with high-prior knowledge, 2 or 3 students with middle-prior knowledge and low-prior knowledge

- The study was conducted in 2 lessons. In lesson 1, students were given 4 geometry questions and 2 geometry problems in lesson 2. Students are placed in groups based on prior knowledge and gender diferences.

Table 1: Research Subject

Prior Knowledge	Lesson 1	Lesson 2	Pairing Male	Pairing Female
High	AZ	FF	AZ - FF	AS - AGM
	AS	AGM		
Middle	RH	RA	RH - RA	NDN - MDS
	NDN	MDS		
Low	WRS	OZ	WRS - OZ	RAS - ES
	RAS	ES		

Lesson study design

Defining the problem
- conducting a student's prior knowledge test
- find out the constraints related to the prerequisite

planning lesson 1
- making lesson plans for lesson 1 with model lecturers

Open Lesson
- Teaching of model lecturers
- Observer observing lessons

Discussion
- evaluating lessons and observations then reflected in discuss

Planing Lesson 2
- revising lessons, making lesson plans for lesson 2

Open Lesson
- Like previous step

Data Collection

- Apply the observation technique in the form of video
- Twelve students were randomly selected to be observed and interviewed, with 6 students in each lesson. These 6 students consisted of 3 male and female pairs representing high, middle, and low-prior knowledge
- Students give feedback on how they complete problem solving tasks through forms of semi-structured interviews and tests of problem solving skills.

- Six students were selected from each different lesson study class based on ability and gender.
- To gather opinions and feedback from students, interviews were conducted in a closed room.
- Students present in a paired room without a model lecturer there. The students entered in pairs to provide feedback with each partner consisting of one student with the same gender and prior knowledge from each research class

Aim of research lessons

To explore, using lesson study, advanced mathematical thinking skills to solve problems. The description describes pedagogical practices that might effectively develop students' ability to solve problems in terms of advanced thinking abilities and gender differences

Lesson 1

- All groups seemed very enthusiastic to begin the task. Students desire to get an answer as quickly as possible with less time spent considering problems, makes many groups of students have difficulty in completing their assignments. This is **something that needs to be changed in lesson 2**.

- Some of students solve problems with confidence and arrive at the answers, but they find that their answers are wrong. Some student claimed that they were not sure whether the answer was correct

- There were students arrived at their answers, but could not explain the answers and how they found the answers.

- They were also asked to answer our questions such as footage of our interview with a female student with low prior knowledge, "Are you sure when answering the questions?" She said "I feel completely sure that my answers are correct when I can solve it myself or the questions have been learned previously. Even though I somehow feel doubtful about the result when my answers are being checked.

Adaptation

Students want to find the answer immediately, but at the beginning of each assignment each group is very quick to start talking about answers rather than ways to get answers. So, we decided to give the group five minutes extra time to ask the questions needed to solve the problem before starting solve it

To overcome this, we reduce the assignments students must do in lesson 2, so that there are only 2 problems. This adjustment is done as an antisifation to the fact that many students do not know where to start in solving problems.

Lesson 2

- Providing special time for students to discuss matters related to the problem with their groups at the beginning of solving the problems is effective in making them think carefully about the information they need and what they have to solve the problem. This is very important to do in learning geometry

- The student often misunderstand problems, so their failure to find solutions. However, overall it seems that students need to be encouraged to spend more time enjoying problem solving.

- The student group with the distribution of more male students, turned out to be more competitive with each other. As a result, they are in a hurry to find a solution. This way makes them lose the opportunity to understand the problem well. The worst result is that they make mistakes in solving problems. These students should be mathematically able to solve problems properly. These students are accustomed to working on algorithmic problems, which are problems that they can solve quickly. The most important thing is the need for emphasis that they must be more thorough and careful in solving problems.

The group of students with a greater distribution of female students is far more thorough. They expand their problems by asking a few questions, and in this way they get additional guidance from some of their group friends. The impact is they can solve the problem correctly
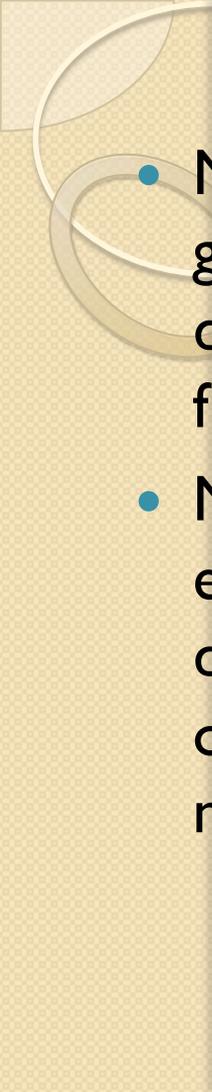
- Male students with high-prior knowledge tend to dominate in group discussions. Female students with high-prior knowledge do not dominate the group; most of them patiently help their friends who are having difficulties.

- Male and female students with middle-prior knowledge are easier to communicate with their friends. When they have difficulties in soving problems, they quickly ask questions with other friends. Similarly, when they know something, they do not hesitate to help their friends.

Conversely, female students or male student with low-prior knowledge tend to withdraw. Therefore, lecturers should give different scaffolding to students with different prior knowledge and gender

Conclusion

In general, students are encouraged to find the answers as quickly as possible, but there are some of them more careful in solving problems. This is what affects their ability to solve problems.

Students are too worried if they do not immediately find the answer, so they do not give enough time to examine the problem, determine what they know and what they need to answer the problem, do not make a conjecture, and the most fatal is that they do not find the answer. Unfortunately, this is the nature of mathematics that appears to students.

- All groups with different abilities get different benefits when solving problems. For students with high-prior knowledge, they need to be encouraged not to rush to solve problems, because the formulation of questions and exploration of problems is something that needs to be done.

- For students with middle-prior knowledge do have sufficient ability to solve problems but they sometimes need guidance to arrive at problem answers. Some observations concluded that they did have good skills to solve problems after some guidance, they began asking good questions and exploring problems deeper.

Students with low-prior knowledge need to get more guidance, because when they are asked to solve problems that are not appropriate for their level of mathematical ability, they will withdraw from the problem

- The ability of male and female students in solving problems **is not necessarily different**. However, there are gender differences with respect to the habits and methods used to overcome the problem. Male students, eventhough they have better abilities, because they are often in a hurry, so in solving problems often fail. While female students are generally more patient and conscientious, despite having the usual ability to solve problems correctly. When groups are diverse, there is more discussion relative to what they have to do to solve the problem.

Students' ability to solve problems seems to vary due to many things including prior knowledge, gender differences, motivation, habits, self confidence and so on. However, problem solving is a very important aspect for mathematics education and its development requires more effort from the teacher

Thank You