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The Locating-Chromatic Number for Certain Operation of Generalized Petersen Graphs $sP(4,2)$

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Abstract. The locating-chromatic number of a graph combined two graph concept, coloring vertices and partition dimension of a graph. The locating-chromatic number, denoted by $\chi_L(G)$, is the smallest k such that G has a locating k-coloring. In this paper, we discuss the locating-chromatic number for certain operation of generalized Petersen graphs $sP(4,2)$.

1. Introduction

Chartrand et *al*. [1] in 2002 introduced the locating-chromatic number of a graph, with derived two graph concept, coloring vertices and partition dimension of a graph. Let $G = (V, E)$ be a connected graph and c be a proper k-coloring of G with color $1,2, \ldots, k$. Let $\Pi = \{C_1, C_2, \ldots, C_k\}$ be a partition of $V(G)$ which is induced by coloring c. The color code $c_\Pi(v)$ of v is the ordered k-tuple $(d(v, C_1), d(v, C_2),\ldots,d(v, C_k))$ where $d(v, C_i)$ = min $\{d(v, x)|x \in C_i\}$ for any i. If all distinct vertices of G have distinct color codes, then c is called k-locating coloring of G. The locating-chromatic number, denoted by $\chi_L(G)$, is the smallest k such that G has a locating k-coloring.

In 2003, Chartrand et *al.* [2] successed in constructing $n \geq 5$ tree graphs with locating-chromatic numbers ranging from 3 to n, except (n – *1*). Behtoe and Omoomi [3] found the locating-chromatic numbers on the Kneser graph. Furthermore, Baskoro and Purwasih [4] found the locating chromatic number for corona product of graphs. Next, Asmiati [5] determined the locating chromatic number of banana tree graph and Asmiati et *al*. [6] for amalgamation of stars graphs. Asmati et *al*. [7] also found the locating chromatic number of firecracker graphs and Syofyan et *al*. [8] for lobster graph.

Specially for non-homogenous tree graph in 2014, Asmiati [9] determined the locating-chromatic number of non-homogeneous amalgamation of stars, then Asmiati [10] for caterpillar graphs and non-homogenous firecracker graphs. In 2017, Asmiati et *a*l. [11] determined some generalized Petersen graphs $P(n, 1)$ having locating-chromatic number 4 for odd $n \geq 3$ or 5 for even $n \geq 4$.

The generalized Petersen graph $P(n,k)$, $n \geq 3$ and $1 \leq k \leq \lfloor (n-1)/2 \rfloor$, consists of an outer *n*-cycle $u_1, u_2, \dots, u_n$, a set of *n* spokes u_i, v_i, $1 \leq i \leq n$, and *n* edges v_i, v_{i+k}, $1 \leq i \leq n$, with indices taken modulo *n*. The generalized Petersen graph was introduced by Watkins in [12].

To define the generalized Petersen graph $sP(4,2)$, suppose there are *s*generalized Petersen graph $P(4,2)$. Some vertices on the outer cycle u_i , $i = 1,2,3,4$ for the generalized Petersen graph t^{th}, $t = 1,2, \dots, s$, $s \geq 1$ denoted by u_i^t, while some vertices on the inner cycle v_i , $i = 1,2,3,4$ for the generalized Petersen graph t^{th}, $t = 1,2, \dots, s$, $s \geq 1$ denoted by v_i^t. Generalized Petersen graph $sP(4,2)$ obtained from $s \geq 1$ graph $P(4,2)$, which every vertices on the outer cycle u_i^t, $i \in [1,4]$, $t \in [1,s]$ connected by a path $\left(u_i^t u_i^{t+1}\right)$ $t = 1,2, \dots, s-1$, $s \geq 2$.

Some researchers have determined the locating-chromatic number for certain operation. Behtoei and Omoomi [13] obtained locating-chromatic number from the grid, cartesian multiplication for trajectories and complete graphs, and cartesian multiplication of two complete graphs. Furthermore Behtoei and Omoomi [14] determined the locating-chromatic number of the fan graph, wheel and friendship graph for join multiplication of two graphs. Asmiati [15] foundlocating-chromatic number for certain operation of tree. In this paper, we discuss the locating-chromatic number for certain operation of generalized Petersen graphs $sP(4,2)$.

The following theoremsis basic to determine the locating chromatic number of a graph.The set of neighbours of a vertex *s* in G, denoted by $N(s)$.

Theorem 1.1. Chartrand et *al.*[1] *Let c be a locating coloring in a connected graph* G*. If r and s are distinct vertices of G such that d(r,w)=d(s,w) for all* $w \in V(G) - \{r, s\}$*, then* $c(r) \neq c(s)$*. In particular, if x and y are non-adjacent vertices of G such that* $N(x) \neq N(y)$*, then* $c(x) \neq c(y)$*.*

Theorem 1.2. Chartrand et *al.*[1] *The locating chromatic number of a cycle* C_n*, is 3 for odd n and 4 for otherwise.*

2. Results and Discussion

In this section we will discuss the locating chromatic number of$sP(4,2)$.

Theorem 2.1. The locating chromatic number of generalized Petersen graph $sP(4,2)$is 5 for $s \geq 2$.

Proof : First, we determine lower bound of $\chi_L\left(sP(4,2)\right)$ for s ≥ 2. Because generalized Petersen graph $P(4,2)$, for s ≥ 2, contains some even cycles. Then by Theorem 2, $\chi_L\left(sP(4,2)\right) \geq 4$. Next, we will show that $\chi_L\left(sP(4,2)\right) \geq 5$, for s ≥ 2. For a contradiction, suppose that *c* is 4-locating coloring on $sP_{4,1}$for$s \geq 2$. Consider $c\left(u_i^1\right) = i, i = 1,2,3,4$ and $c\left(v_j^1\right) = j, j = 1,2,3,4$ such that $c\left(u_i^1\right) \neq c\left(v_j^1\right)$ for $c\left(u_i^1\right)$ adjacent to$c\left(v_j^1\right)$. Observe that if we assign color 4 for any vertices in u_i^2 or v_i^2, then we have two vertices which have color codes. Therefore, *c* is not locating 4-coloring on $sP(4,2)$. As the result $\chi_L(sP(4,2)) \geq 5$ for $s \geq 2$.

Next, we determine the upper bound of $\chi_L\left(sP(4,2)\right)$ for s ≥ 2. Let *c* be a coloring of generalized Petersen graph $sP(4,2)$ for s ≥ 2. We make the partition of the vertices of $V(sP(4,2))$:

$C_1 = \{u_1^t | \text{for odd } s\} \cup \{u_2^t, v_4^t | \text{for even } s\}$
$C_2 = \{u_2^t, u_4^t | \text{for odd } s\} \cup \{u_3^t, v_1^t | \text{for even } s\}$
$C_3 = \{u_3^t, v_1^t, v_2^t | \text{for odd } s\} \cup \{u_4^t, v_2^t, v_3^t | \text{for even } s\}$
$C_4 = \{v_3^t | \text{for odd } s\} \cup \{u_1^t | \text{untuk } s \text{genap}\} \cup \{v_4^t | \text{for odd } s \geq 3\}$
$C_5 = \{v_4^1\}$

Therefore the color codes of all the vertices of G are :

(a) $C_1 = \{u_1^t | \text{for odd } s \} \cup \{u_2^t, v_4^t | \text{for even } s \}$

For odd *s*, the color codes of $sP(4,2)$ are:

$$c_\Pi(u_1^t) = \begin{cases} 0 & , \text{ for } 1^{st} \text{ component} \\ 1 & , \text{ for } 2^{nd}, 3^{rd} \text{ and } 4^{th} \text{ component} \\ s+1 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

For even s, the color codes of $sP(4,2)$ are:

$$c_\Pi(u_2^t)=\begin{cases} 0 & , \text{ for } 1^{st} \text{ component} \\ 1 & , \text{ for } 2^{nd}, 3^{rd} \text{ and } 4^{th} \text{ component} \\ s+1 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

$$c_\Pi(v_4^t)=\begin{cases} 0 & , \text{ for } 1^{st} \text{ component} \\ 2 & , \text{ for } 2^{nd} \text{and } 4^{th} \text{ component} \\ 1 & , \text{ for } 3^{rd} \text{ component} \\ s+1 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

(b) $C_2=\{u_2^t,u_4^t|\text{for odd } s\}\cup\{u_3^t,v_1^t|\text{for even } s\}$
For odd s the color codes of $sP(4,2)$ are:

$$c_\Pi(u_2^t)=\begin{cases} 1 & , \text{ for } 1^{st} \text{and } 3^{rd} \text{component} \\ 0 & , \text{ for } 2^{nd} \text{ component} \\ 4 & , \text{ for } 4^{th} \text{ component} \\ s+1 & , \text{ for} 5^{th} \text{ component} \end{cases}$$

$$c_\Pi(u_4^1)=\begin{cases} 1 & , \text{ for } 1^{st}, 3^{rd} \text{ and } 5^{th} \text{ component} \\ 0 & , \text{ for } 2^{nd} \text{ component} \\ 2 & , \text{ for } 4^{th} \text{ component} \end{cases}$$

For odd $s\geq 3$, the color codes of $sP(4,2)$ are:

$$c_\Pi(u_4^t)=\begin{cases} 1 & , \text{ for } 1^{st}, 3^{rd} \text{ and } 4^{th} \text{ component} \\ 0 & , \text{ for } 2^{nd} \text{ component} \\ s & , \text{ for } 5^{th} \text{ component} \end{cases}$$

For even s, the color codes of $sP(4,2)$are:

$$c_\Pi(u_3^t)=\begin{cases} 1 & , \text{ for } 1^{st} \text{and } 3^{rd} \text{component} \\ 0 & , \text{ for} 2^{nd} \text{ component} \\ 2 & , \text{ for } 4^{th} \text{ component} \\ s+1 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

$$c_\Pi(v_1^t)=\begin{cases} 2 & , \text{ for } 1^{st} \text{ component} \\ 0 & , \text{ for } 2^{nd} \text{ component} \\ 1 & , \text{ for } 3^{rd} \text{ and } 4^{th} \text{ component} \\ s+2 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

(c) $C_3=\{u_3^t,v_1^t,v_2^t|\text{for odd } s\}\cup\{u_4^t,v_2^t,v_3^t|\text{for even } s\}$.
For odd s, the color codes of $sP(4,2)$ are:

$$c_\Pi(u_3^t)=\begin{cases} 2 & , \text{ for } 1^{st} \text{ component} \\ 1 & , \text{ for } 2^{nd} \text{ and } 4^{th} \text{ component} \\ 0 & , \text{ for } 3^{rd} \text{ component} \\ s+1 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

$$c_\Pi(v_1^t)=\begin{cases} 1 & , \text{ for } 1^{st} \text{and} 4^{th} \text{ component} \\ 2 & , \text{ for } 2^{nd} \text{ component} \\ 0 & , \text{ for } 3^{rd} \text{ component} \\ s+2 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

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$$c_\Pi(v_2^1) = \begin{cases} 2 & , \text{ for } 1^{st} \text{ component} \\ 1 & , \text{ for } 2^{nd} \text{ and } 5^{th} \text{ component} \\ 0 & , \text{ for } 3^{rd} \text{ component} \\ 3 & , \text{ for } 4^{th} \text{ component} \end{cases}$$

For odd $s \geq 3$ the color codes of $sP(4,2)$ are:

$$c_\Pi(v_2^t) = \begin{cases} 2 & , \text{ for } 1^{st} \text{and} 4^{th} \text{ component} \\ 1 & , \text{ for } 2^{nd} \text{ component} \\ 0 & , \text{ for } 3^{rd} \text{ component} \\ s+2 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

For even s the color codes of $sP(4,2)$ are:

$$c_\Pi(u_4^t) = \begin{cases} 1 & , \text{ for } 1^{st}, 2^{nd} \text{ and } 4^{th} \text{ component} \\ 0 & , \text{ for } 3^{rd} \text{ component} \\ s & , \text{ for } 5^{th} \text{ component} \end{cases}$$

$$c_\Pi(v_2^t) = \begin{cases} 1 & , \text{ for } 1^{st} \text{ component} \\ 2 & , \text{ for } 2^{nd} \text{ and } 4^{th} \text{ component} \\ 0 & , \text{ for } 3^{rd} \text{ component} \\ s+2 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

$$c_\Pi(v_3^t) = \begin{cases} 2 & , \text{ for } 1^{st} \text{and } 4^{th} \text{ component} \\ 1 & , \text{ for } 2^{nd} \text{ component} \\ 0 & , \text{ for } 3^{rd} \text{ component} \\ s+2 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

(d) $C_4 = \{v_3^t | \text{for odd } s\} \cup \{u_1^t | \text{for even } s\} \cup \{v_4^t | \text{for odd } s \geq 3\}$

For odd s the color codes of $sP(4,2)$ are:

$$c_\Pi(v_3^t) = \begin{cases} 2 & , \text{ for } 1^{st} \text{and } 2^{nd} \text{component} \\ 1 & , \text{ for } 3^{rd} \text{ component} \\ 0 & , \text{ for } 4^{th} \text{ component} \\ s+2 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

For odd $s \geq 3$ the color codes of $sP(4,2)$ are:

$$c_\Pi(v_4^t) = \begin{cases} 2 & , \text{ for } 1^{st} \text{ component} \\ 1 & , \text{ for } 2^{nd} \text{ and } 3^{rd} \text{ component} \\ 0 & , \text{ for } 4^{th} \text{ component} \\ s+1 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

For even s the color codes of $sP(4,2)$ are:

$$c_\Pi(v_4^t) = \begin{cases} 1 & , \text{ for } 1^{st}, 2^{nd} \text{ and } 3^{rd} \text{ component} \\ 0 & , \text{ for } 4^{th} \text{ component} \\ s+1 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

(e) $C_5 = \{v_4^1\}$

$$c_{\Pi}(v_4^1) = \begin{cases} 2 & , \text{ for } 1^{st} \text{ component} \\ 1 & , \text{ for } 2^{nd} \text{ and } 3^{rd} \text{ component} \\ 3 & , \text{ for } 4^{th} \text{ component} \\ 0 & , \text{ for } 5^{th} \text{ component} \end{cases}$$

Since all the vertices have different color codes, c is a locating coloring of generalized Petersen graphs $sP(4,2)$, so $\chi_L(sP(4,2)) = 5$, for even $s \geq 2$. □

In figure 1 is illustrated a locating coloring of generalized Petersen graphs $4P(4,2)$ with the locating chromatic number 5.

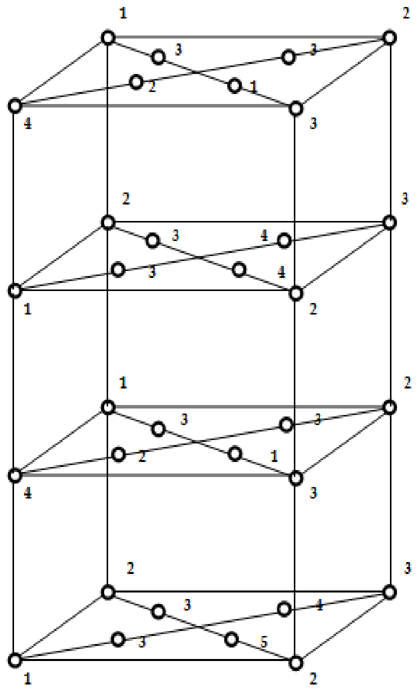

Figure 1. A minimum locating coloring of $4P(4,2)$

3. Conclusion

Based on the results, locating chromatic number of generalized Petersen graph $sP(4,2)$ is 5 for $s \geq 2$.

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