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$T[[S]]$-NOETHERIAN PROPERTY ON GENERALIZED POWER SERIES MODULES

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Abstract

In this paper, we study the sufficient conditions on a ring R, a multiplicative set $T \subseteq R$, a strictly ordered monoid $(S, \leq)$ and an R-module M such that the generalized power series module $M[[S]]$ is a $T[[S]]$-Noetherian $R[[S]]$-module, where $T[[S]]$ is a multiplicative subset of generalized power series ring $R[[S]]$.

1. Introduction

Anderson and Dumitrescu [2] introduced the definition of T-Noetherian rings and modules. For any multiplicatively closed subset T of a ring R, a

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ring R is called *T-Noetherian* if each ideal of R is T-finite, i.e., if $TI \subseteq J \subseteq I$ for some finitely generated ideal J of R and some $t \in T$. An R-module M is called an *T-Noetherian module* if for each submodule N of M, there exist an element $t \in T$ and a finitely generated submodule F of M such that $Nt \subseteq F \subseteq N$. Some properties of T-Noetherian modules are studied by Baeck et al. [3]. Varadarajan [4] constructs the generalized power series modules (GPSM) $M[[S]]$, which is a module over generalized power series rings (GPSR) $R[[S]]$ constructed by Ribenboim [6]. Moreover, Varadarajan determined the necessary and sufficient conditions for GPSM $M[[S]]$ to be a Noetherian module, which strengthens earlier results of Ribenboim.

In this paper, we investigate the sufficient conditions for $R[[S]]$-module $M[[S]]$ to be $T[[S]]$-Noetherian.

2. GPSR and GPSM

In this section, we recall the construction of GPSR and GPSM as follows from Ribenboim [6] and Varadarajan [4].

Regarding ordered sets, strictly ordered monoids, Artinian and narrow sets, we will be following the terminology in [6] and [7].

Let $(S, \leq)$ be a strictly ordered monoid, R be a commutative ring with an identity element and M be an R-module. Let $R^S = \{f \mid f : S \to R\}$ and

$$R[[S]] = \{f \in R^S \mid \operatorname{supp}(f) \text{ is Artinian and narrow}\},$$

where $\operatorname{supp}(f) = \{s \in S \mid f(s) \neq 0\}$.

For any $f, g \in R[[S]]$, $\operatorname{supp}(f + g) \subseteq \operatorname{supp}(f) \cup \operatorname{supp}(g)$, $\operatorname{supp}(-f) = \operatorname{supp}(f)$, and $\operatorname{supp}(fg) \subseteq \operatorname{supp}(f) + \operatorname{supp}(g)$. Therefore, under pointwise addition and convolution multiplication defined by

$$(fg)(s) = \sum_{(x, y) \in \chi_s(f, g)} f(x)g(y), \tag{2.1}$$

for all $f, g \in R[[S]]$, where

$$\chi_s(f, g) = \{(x, y) \in \text{supp}(f) \times \text{supp}(g) \mid xy = s\}$$

is finite, $R[[S]]$ becomes a ring which is known as generalized power series ring (GPSR).

Next, let $M^S = \{\alpha \mid \alpha : S \to M\}$ and

$$M[[S]] = \{\alpha \in M^S \mid \text{supp}(\alpha) \text{ is Artinian and narrow}\},$$

where $\text{supp}(\alpha) = \{s \in S \mid \alpha(s) \neq 0\}$. For any $\alpha, \beta \in M[[S]]$, $\text{supp}(\alpha + \beta) \subseteq \text{supp}(\alpha) \cup \text{supp}(\beta)$, $\text{supp}(-\alpha) = \text{supp}(\beta)$, and $\text{supp}(\alpha\beta) \subseteq \text{supp}(\alpha) + \text{supp}(\beta)$. Therefore, under pointwise addition and scalar multiplication defined by

$$(f\alpha)(s) = \sum_{(x, y) \in \chi_s(f, \alpha)} f(x)\alpha(y), \tag{2.2}$$

for all $f \in R[[S]]$ and $\alpha \in M[[S]]$, where

$$\chi_s(f, \alpha) = \{(x, y) \in \text{supp}(f) \times \text{supp}(\alpha) \mid xy = s\}$$

is finite, $M[[S]]$ acquires the structure of an $R[[S]]$-module. Next, this module is called *generalized power series module* (*GPSM*).

For any $r \in R$ and any $s \in S$, we associate the maps $c_r, e_s \in R[[S]]$, defined by

$$c_r(t) = \begin{cases} r & \text{if } t = 1 \\ 0 & \text{if } t \neq 1 \end{cases} \tag{2.3}$$

and

$$e_s(t) = \begin{cases} 1 & \text{if } t = s \\ 0 & \text{if } t \neq s. \end{cases} \tag{2.4}$$

For any $m \in M$ and any $s \in S$, we define $d_m^s(t) \in M[[S]]$ by

$$d_m^s(t) = \begin{cases} m & \text{if } t = s \\ 0 & \text{if } t \neq s. \end{cases} \tag{2.5}$$

Then, it is clear that $r \mapsto c_r$ is a ring embedding of R into $R[[S]]$, and $s \mapsto e_s$ is a monoid embedding of S into the multiplicative monoid of the ring $R[[S]]$, and also $m \mapsto d_m^0$ is a module embedding of M into $M[[S]]$.

3. Sufficient Conditions for GPSM $M[[S]]$ to be $T[[S]]$-Noetherian

In this section we give the sufficient conditions for GPSM $M[[S]]$ to be $T[[S]]$-Noetherian module. For any ring R and for some $n \geq 1$, we denote $R \oplus R \oplus \cdots \oplus R$ (n factors), by $\oplus R^{(n)}$. The necessary and sufficient condition for R-module M to be a finitely generated module is given by the following lemma.

Lemma 3.1 (See [2, Lemma 3]). *M is finitely generated R-module if and only if it is isomorphic to a quotient of* $\oplus R^{(n)}$ *for some* $n > 0$.

For any subset N of an R-module M, we define

$$N[[S]] = \{\alpha \in M[[S]] \mid \alpha(s) \in N;\ \forall s \in S\}.$$

The following lemma shows that $N[[S]]$ is an $R[[S]]$-submodule of $M[[S]]$.

Lemma 3.2. *Let M be an R-module and* $M[[S]]$ *be an* $R[[S]]$*-module. Then,* $N[[S]]$ *is an* $R[[S]]$*-submodule of* $M[[S]]$.

Proof. For any $\alpha, \beta \in N[[S]]$ and $f, g \in R[[S]]$, we will show that $\alpha f + \beta g \in N[[S]]$. In other words, it is enough to show $(\alpha f + \beta g)(s) \in N$ for every $s \in S$. For any $\alpha \in N[[S]]$, $f \in R[[S]]$, and for every $s \in S$, $(\alpha f)(s) = \sum_{uv=s} \alpha(u) f(v)$. Since N is an R-submodule of M and $\alpha \in$

$N[[S]],\ \alpha(u)f(v) \in N$. Hence, $(\alpha f)(s) \in N$ for every $s \in S$. In similar way, for any $\beta \in N[[S]],\ g \in R[[S]]$, and for every $s \in S$, we obtain $(\beta g)(s) \in N$. Therefore, $(\alpha f + \beta g)(s) \in N$ for every $s \in S$. So, $N[[S]]$ is an $R[[S]]$-submodule of $M[[S]]$. □

The following proposition shows that a GPSM with coefficients from M/N is equivalent with a module factor of GPSM $M[[S]]/N[[S]]$.

Proposition 3.3. *Let M be an R-module and $M[[S]]$ be an $R[[S]]$-module. If N is an R-submodule of M, then* $(M/N)[[S]] \cong M[[S]]/N[[S]]$.

Proof. For any R-submodule N of M, let p_N be a natural projection. Next, we define a map $\varphi : M[[S]] \to (M/N)[[S]]$, by

$$\alpha \mapsto \overline{\alpha} = p_N \circ \alpha,$$

for every $\alpha \in M[[S]]$. It is easy to show that $\text{supp}(\overline{\alpha})$ is Artinian and narrow, which is $\overline{\alpha} \in (M/N)[[S]]$.

For any $\overline{\alpha} \in (M/N)[[S]]$, there exist $\alpha \in M[[S]]$. Then φ is surjective. Therefore, $\text{Im}(\varphi) = (M/N)[[S]]$. Next, if $\varphi(\alpha) = \overline{0}$ for any $\alpha \in M[[S]]$, then $\overline{\alpha} = p_N \circ \alpha = \overline{0}$. Therefore, $\alpha(s) \in N$ for every $s \in S$. So, $Ker(\varphi) = N[[S]]$. Hence, based on the fundamental isomorphism theorem of modules, we obtain $(M/N)[[S]] \cong M[[S]]/N[[S]]$. □

The following proposition shows that the GPSR over $\oplus R^{(n)}$ is isomorphic to the direct sum of GPSR $R[[S]] \oplus \cdots \oplus R[[S]]$ (n factors).

Proposition 3.4. *Let R be a ring*, $(S, \leq)$ *be a strictly ordered monoid*, $R[[S]]$ *be a GPSR and* $n \geq 1$. *Then* $(\oplus R^{(n)})[[S]] \cong \oplus(R[[S]])^{(n)}$.

Proof. It is a special case of [2, Proposition 9], by choosing $R_i = R$ for every i and a monoid homomorphism $\omega^{(i)}(s) = id_{R_i}$ for every $s \in S$. □

The sufficient condition for $R[[S]]$-module $M[[S]]$ to be finitely generated module is given by the following proposition.

Proposition 3.5. *Let M be an R-module and* $M[[S]]$ *be an* $R[[S]]$-*module. If M is finitely generated, then so is* $M[[S]]$.

Proof. Based on Lemma 3.1, it is enough to show $M[[S]] = \oplus (R[[S]])^{(n)}/N,$ for some submodule N of $\oplus (R[[S]])^{(n)}$. Since M is finitely generated, by Lemma 3.1, $M \cong \oplus R^{(n)}/K,$ for some submodule K of $\oplus R^{(n)}$. Since K is a submodule of $\oplus R^{(n)},$ based on Lemma 3.2 $K[[S]]$ is a submodule of $(\oplus R^{(n)})[[S]]$. Furthermore, base on Proposition 3.4, we have $K[[S]]$ is a submodule of $\oplus (R[[S]])^{(n)}$. Hence, we can choose $N = K[[S]]$.

Now, we will show, $(\oplus R^{(n)}/K)[[S]] \cong \oplus (R[[S]])^{(n)}/K[[S]]$. By using Proposition 3.3, we get

$$(\oplus R^{(n)}/K)[[S]] \cong (\oplus R^{(n)})[[S]]/K[[S]].$$

Furthermore, by using Proposition 3.4, we get $(\oplus R^{(n)})[[S]] \cong \oplus (R[[S]])^{(n)}$.

So, $(\oplus R^{(n)}/K)[[S]] \cong (\oplus R^{(n)})[[S]]/K[[S]] \cong \oplus (R[[S]])^{(n)}/K[[S]]$. In other words, $M[[S]] \cong \oplus (R[[S]])^{(n)}/N$, or $M[[S]]$ is finitely generated as an $R[[S]]$-module. □

Next, for any subset T of a ring R, we define the set

$$T[[S]] = \{f \in R[[S]] \mid f(s) \in T;\ \forall s \in \operatorname{supp}(f)\}.$$

It is clear that $T[[S]] \subseteq R[[S]]$. The sufficient conditions of $T[[S]]$ to be a multiplicatively closed subset of GPSR $R[[S]]$ are given by the following lemma.

Lemma 3.6. *Let R be a ring, T be a multiplicative subset of R,* $(S, \leq)$ *be a strictly ordered monoid, and* $R[[S]]$ *be a GPSR. If T is additively closed, then* $T[[S]]$ *is a multiplicative subset of* $R[[S]]$.

Proof. For any $f, g \in T[[S]]$, we will show that $fg \in T[[S]]$. Based on the convolution multiplication in equation (2.1), for any $s \in \text{supp}(fg)$ we obtain $(fg)(s) = \sum_{xy=s} f(x)g(y)$. Since $T \subseteq R$ is multiplicatively and additively closed, we have $\sum_{xy=s} f(x)g(y) \in T$ for every $s \in \text{supp}(fg)$. In other words, $fg \in T[[S]]$. Thus, $T[[S]]$ is a multiplicatively closed subset of $R[[S]]$. □

From equation (2.3), it clear that R is isomorphic to the subring $\{c_r \mid r \in R\}$ of $R[[S]]$. Thus, if *T* is a multiplicative subset of *R*, then $C(T) = \{c_t \mid t \in T\}$ is a multiplicative subset of $R[[S]]$. Then, it is clear that $T \cong C(T) \subseteq T[[S]]$.

We recall a multiplicative subset *T* of a ring *R* is anti-Archimedean if $\bigcap_{n \geq 1} t^n RT \neq \varnothing$, for every $t \in T$. The sufficient conditions for $R[[S]]$-module $M[[S]]$ to be a $T[[S]]$-Noetherian module are given by the following theorems.

Theorem 3.7. *Let* $T \subseteq R$ *be an additive and multiplicative set with anti-Archimedean property, R be a T-Noetherian ring,* $S = \mathbb{N} \cup \{0\}$ *be a strictly ordered monoid with a trivial order* $\leq$, *and M be a finitely generated R-module. Then,* $R[[S]]$*-module* $M[[S]]$ *is* $T[[S]]$*-Noetherian.*

Proof. Since R is *T*-Noetherian, $T \subseteq R$ is an anti-Archimedean multiplicative subset of *R* and $S = \mathbb{N} \cup \{0\}$ is a strictly ordered monoid with a trivial order $\leq$, based on [2, Proposition 9], $R[[S]]$ is *T*-Noetherian. Next, based on Proposition 3.5, $M[[S]]$ is finitely generated as an $R[[S]]$-module. Then, based on [3, Lemma 2.14(4)], we get $M[[S]]$ is *T*-Noetherian.

Furthermore, based on Lemma 3.6., $T[[S]]$ is a multiplicative subset of $R[[S]]$.

Since $T \subseteq T[[S]]$, based on [3, Remark 2.11(2)], we have $M[[S]]$ is $T[[S]]$-Noetherian. □

Theorem 3.8. *Let $T \subseteq R$ be an additive and multiplicative set with anti-Archimedean property, R be a T-Noetherian ring, $S = \mathbb{Z}$ be a strictly ordered monoid with a trivial order $\leq$, and M be a finitely generated R-module. Then, $R[[S]]$-module $M[[S]]$ is $T[[S]]$-Noetherian.*

Proof. Since R is T-Noetherian, $T \subseteq R$ is an anti-Archimedean multiplicative subset of R and $S = \mathbb{Z}$ is a strictly ordered monoid with a trivial order $\leq$, based on [3, Proposition 3.4], $R[[S]]$ is T-Noetherian. Next, based on Proposition 3.5, $M[[S]]$ is finitely generated as an $R[[S]]$-module. Then, based on [3, Lemma 2.14(4)], we get $M[[S]]$ is T-Noetherian. Furthermore, based on Lemma 3.6, $T[[S]]$ is a multiplicative subset of $R[[S]]$. Since $T \subseteq T[[S]]$, based on [3, Remark 2.11 (2)], we have $M[[S]]$ is $T[[S]]$-Noetherian. □

Theorem 3.9. *Let $T \subseteq R$ be an additive and multiplicative set with anti-Archimedean property consisting of nonzero divisor, R be a T-Noetherian ring, $S = \mathbb{N} \cup \{0\}$ be a strictly ordered monoid with a usual order $\leq$, and M be a finitely generated R-module. Then, $R[[S]]$-module $M[[S]]$ is $T[[S]]$-Noetherian.*

Proof. Since R is T-Noetherian, $T \subseteq R$ is an anti-Archimedean multiplicative subset of R consisting of nonzero divisor and $S = \mathbb{N} \cup \{0\}$ is a strictly ordered monoid with a usual order $\leq$, based on [2, Proposition 10], $R[[S]]$ is T-Noetherian. Next, based on Proposition 3.5, $M[[S]]$ is finitely generated as an $R[[S]]$-module. Then, based on [3, Lemma 2.14(4)], we get $M[[S]]$ is T-Noetherian. Furthermore, based on Lemma 3.6, $T[[S]]$ is a multiplicative subset of $R[[S]]$. Since $T \subseteq T[[S]]$, based on [3, Remark 2.11 (2)], we have $M[[S]]$ is $T[[S]]$-Noetherian. □

Theorem 3.10. *Let $T \subseteq R$ be an additive and multiplicative set with anti-Archimedean property consisting of nonzero divisor, R be a T-Noetherian ring, $S = \mathbb{Z}$ be a strictly ordered monoid with a usual order $\leq$, and M be a finitely generated R-module. Then, $R[[S]]$-module $M[[S]]$ is $T[[S]]$-Noetherian.*

Proof. Since R is T-Noetherian, $T \subseteq R$ is an anti-Archimedean multiplicative subset of R consisting of nonzero divisor and $S = \mathbb{Z}$ is a strictly ordered monoid with a usual order $\leq$, based on [5, Theorem 3.1], $R[[S]]$ is T-Noetherian. Next, based on Proposition 3.5, $M[[S]]$ is finitely generated as an $R[[S]]$-module. Then, based on [3, Lemma 2.14(4)], we get $M[[S]]$ is T-Noetherian. Furthermore, based on Lemma 3.6, $T[[S]]$ is a multiplicative subset of $R[[S]]$. Since $T \subseteq T[[S]]$, based on [3, Remark 2.11(2)], we have $M[[S]]$ is $T[[S]]$-Noetherian. □

Theorem 3.11. *Let $T \subseteq R$ be an additive and multiplicative set with anti-Archimedean property consisting of nonzero divisor, R be a T-Noetherian ring, S be a finitely generated strictly ordered monoid with a positive order $\leq$, and M be a finitely generated R-module. Then, $R[[S]]$-module $M[[S]]$ is $T[[S]]$-Noetherian.*

Proof. Since R is T-Noetherian, $T \subseteq R$ is an anti-Archimedean multiplicative subset of R consisting of nonzero divisor and S is a finitely generated strictly ordered monoid with a positive order $\leq$, based on [5, Theorem 2.3], $R[[S]]$ is T-Noetherian. Next, based on Proposition 3.5, $M[[S]]$ is a finitely generated as an $R[[S]]$-module. Then, based on [3, Lemma 2.14(4)], we get $M[[S]]$ is T-Noetherian. Furthermore, based on Lemma 3.6, $T[[S]]$ is a multiplicative subset of $R[[S]]$. Since $T \subseteq T[[S]]$, based on [3, Remark 2.11(2)], we have $M[[S]]$ is $T[[S]]$-Noetherian. □

Theorem 3.12. *Let R be a ring*, $T \subseteq R$ *be a multiplicative set*, $(S, \leq)$ *be a strictly ordered monoid*, *and M be an R-module*. *If R is Noetherian*, *T is additively closed*, $(S, \leq)$ *is narrow*, *S is cancelative and torsion-free*, $\exists s_1, ..., s_n \in S \backslash G(S)$ *such that* $S = \langle s_1, ..., s_n \rangle + G(S)$, *and M is finitely generated*, *then* $R[[S]]$*-module* $M[[S]]$ *is* $T[[S]]$*-Noetherian*.

Proof. Based on [7, Theorem 5.5], $R[[S]]$ is Noetherian. Therefore, by the definition, $R[[S]]$ is *T*-Noetherian for any multiplicative subset *T* of *R*. Next, based on Proposition 3.5, $M[[S]]$ is finitely generated as an $R[[S]]$-module. Then, based on [3, Lemma 2.14(4)], we get $M[[S]]$ is *T*-Noetherian. Furthermore, based on Lemma 3.6, $T[[S]]$ is a multiplicative subset of $R[[S]]$. Since $T \subseteq T[[S]]$, based on [3, Remark 2.11(2)], we have $M[[S]]$ is $T[[S]]$-Noetherian. □

Theorem 3.13. *Let R be a ring*, $T \subseteq R$ *be a multiplicative set*, $(S, \leq)$ *be a strictly ordered monoid*, *and M be an R-module*. *If T is additively closed*, $(S, \leq)$ *is narrow*, *S is cancelative and torsion-free*, $\exists s_1, ..., s_n \in S \backslash G(S)$ *such that* $S = \langle s_1, ..., s_n \rangle + G(S)$, *M is Noetherian*, *and* $N = \{x \in M \mid Rx \subseteq N\}$ *for every submodule N of M*, *then* $R[[S]]$*-module* $M[[S]]$ *is* $T[[S]]$*-Noetherian*.

Proof. Based on [4, Theorem 4.6], $M[[S]]$ is Noetherian as an $R[[S]]$-module. Therefore, by the definition, $M[[S]]$ is *T*-Noetherian for any multiplicative subset *T* of *R*. Furthermore, based on Lemma 3.6, $T[[S]]$ is a multiplicative subset of $R[[S]]$. Since $T \subseteq T[[S]]$, based on [3, Remark 2.11(2)], we have $M[[S]]$ is $T[[S]]$-Noetherian. □

4. Conclusions

In this section, we give the conclusion of the main results of this paper. These conclusions, we present in the following table:

Table 1. Sufficient conditions for GPSM $M[[S]]$ to be $T[[S]]$-Noetherian

<table>
<tr><th colspan="5">Assumptions</th></tr>
<tr><th>Ring R</th><th>Multiplicatively closed set $T \subseteq R$</th><th>Strictly order monoid $(S, \leq)$</th><th>R-module M</th><th>Theorem</th></tr>
<tr><td rowspan="5">T-Noetherian</td><td rowspan="2">▪ anti-Archimedean
▪ additively closed</td><td>▪ $S = \mathbb{N} \cup \{0\}$
▪ trivial $\leq$</td><td rowspan="6">finitely generated</td><td>3.7</td></tr>
<tr><td>▪ $S = \mathbb{Z}$
▪ trivial $\leq$</td><td>3.8</td></tr>
<tr><td rowspan="3">▪ anti-Archimedean consisting of nonzero divisor
▪ additively
▪ closed</td><td>▪ $S = \mathbb{N} \cup \{0\}$
▪ usual $\leq$</td><td>3.9</td></tr>
<tr><td>▪ $S = \mathbb{Z}$
▪ usual $\leq$</td><td>3.10</td></tr>
<tr><td>▪ f.g. S
▪ positive $\leq$</td><td>3.11</td></tr>
<tr><td>Noetherian</td><td rowspan="2">Additively closed</td><td>▪ $(S, \leq)$ is narrow
▪ S is cancelative and torsion-free
▪ $\exists s_1, \ldots, s_n \in S \backslash G(S)$ s.t. $S = \langle s_1, \ldots, s_n \rangle + G(S)$</td><td>3.12</td></tr>
<tr><td>Any</td><td>▪ $(S, \leq)$ is narrow
▪ S is cancelative and torsion-free
▪ $\exists s_1, \ldots, s_n \in S \backslash G(S)$, s.t. $S = \langle s_1, \ldots, s_n \rangle + G(S)$</td><td>▪ M is Noetherian
▪ $N = \{x \in M \mid Rx \subseteq N\}$, for every $N \leq M$</td><td>3.13</td></tr>
</table>

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