Modeling Stock Return Data using Asymmetric Volatility Models : A Performance Comparison based on the Akaike Information Criterion and Schwarz Criterion

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Abstract

The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model has been widely used in time series forecasting especially with asymmetric volatility data. As the generalization of autoregressive conditional heteroscedasticity model, GARCH is known to be more flexible to lag structures. Some enhancements of GARCH models were introduced in literatures, among them are Exponential GARCH (EGARCH), Threshold GARCH (TGARCH) and Asymmetric Power GARCH (APGARCH) models. This paper aims to compare the performance of the three enhancements of the asymmetric volatility models by means of applying the three models to estimate real daily stock return volatility data. The presence of leverage effects in empirical series is investigated. Based on the value of Akaike information and Schwarz criterions, the result showed that the best forecasting model for daily stock return data is the APARCH model.

Keywords: Volatility, GARCH, TGARCH, EGARCH, APARCH, AIC and SC.

I. INTRODUCTION

BOX and Jenkins (1976) introduces a time series data forecasting model that is now commonly used in economics and known as the Autoregressive Integrated Moving Average [ARIMA(p,d,q)] model. If no differencing is involved, this model is called an Autoregressive Moving Average [ARMA(p,q)] with p and q retaining their original meaning and no d. The ARIMA model is a linear and symmetric model which is appropiate only for linear and symetric data (Makridakis, 1998). However, one often finds asymmetric volatility time series data to forecast. To resolve such data, Engle (1982) introduced Autoregressive Conditional Heteroscedasticity (ARCH) to model inflation data in the UK which contained asymmetric volatility. This model has been proved suitable for data having asymmetric volatility and short lag structures. The ARCH model was extended to GARCH by Bollerslev (1986) which is more flexible to lag structures. Both models have symmetrical volatility response characteristics to shocks, either positive or negative shocks. Financial data in particular stocks have asymmetric volatility, i.e. different volatility movements against an increase or decrease in the price of an asset (Knight and Satchel, 2007). Some of the models that can also be used to overcome asymmetric volatility problems such are TGARCH, EGARCH and APARCH models.

The TGARCH model has the advantage of measuring the volatility of stock prices with any difference in the effects of positive shocks and negative shocks (Zakoian, 1994). Nelson (1991) developed the EGARCH model for asymmetric models. The APARCH model was developed by Ding *et al*. (1993) used to correct the weakness of the ARCH and GARCH models in capturing the phenomenon of asymmetry.

II. Asymmetric-Garch Family Models

In this section we review the GARCH models preceded by Autoregressive Moving Average (ARMA) and Autoregressive Conditional Heteroscedasticity (ARCH) models. Then we present briefly the three asymmetric- GARCH family models.

A. ARMA Model

ARMA models provide a good forecast of volatility. An ARMA(p,q) model is a combination of AR(p) and MA(q) models and is suitable for univariate time series modeling (Gujarati, 2004; Brockwell and Davis, 2002). The ARMA(p,q) model can be expressed as:

$$y_t = c + \varepsilon_t + \sum_{i=1}^{p} \varphi_i y_{t-1} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}$$

Here the models orders p, q refer to p autoregressive and q moving average terms. This form of model assumes that time series is stationary. In absence of a stationary process, the impact of previous values is non-declining. If a process contains a unite root that is non-stationary, and it cannot be modeled as an ARMA model, it instead has to be modeled as an ARIMA.

B. ARCH Model

The Autoregressive conditional heteroscedasticity model, also known as ARCH, is useful when the data researched is a non-linear character. One approach used is to include a free variable capable of predicting the volatility of the error. This is explained in great detail by Bera and Higgins (1993). According to Engle (1982), this varied range of errors occurs because the error range is not only a function of the free variable but also depends on the extent of the error in the past. In the cross section data, the heteroscedasticity that occur directly related to free variables, so to overcome it only need to do the transformation of regression equation. However in the ARCH model, heteroschedasticity occurs because time series data has high volatility. If a data during a period has a high fluctuation and the error is also high, followed by a period where the fluctuation is low and the error is also low, the error range of the model will depend on the fluctuation of the previous error. If the error range depends on the fluctuation of the quadratic error from some previous period (lag p), then the ARCH model (p) can be expressed in terms of the following equation,

$$\sigma_t^2 = \theta_0 + \theta_1 e_{t-1}^2 + \theta_2 e_{t-2}^2 + \cdots + \theta_p e_{t-p}^2.$$

To check the existence of the effect of asymmetric effect one can use sign bias test. Another way is by looking at the correlation between standard residual squares of ARMA model with GARCH residual standard lag model using cross correlation. If there is a stem that exceeds the standard deviation or is marked by an asterisk, meaning that bad news and good news conditions have an asymmetrical effect on volatility.

C. GARCH Model

If an ARMA model is assumed for the error variance Bollerslev (1986) suggests to use GARCH model. In GARCH model the error range depends not only on the past errors but also on the errors of the past period (Francq and Zakoian, 2010). If the error range is affected by the previous period p error (lag p ARCH element) and the error range q of the previous period (lag q GARCH element), then the GARCH model (p, q) can be expressed as:

$$\sigma_t^2 = \theta_0 + \theta_1 e_{t-1}^2 + \theta_p e_{t-p}^2 + \lambda_1 \sigma_{t-1}^2 + \cdots + \lambda_q \sigma_{t-q}^2.$$

D. Three Extensions of GARCH Models

1) EGARCH Model

The EGARCH model has the following form,

$$Ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{e_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \lambda \left[\frac{|e_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right],$$

where ω, β, γ and λ are the estimated parameters. $Ln\,\sigma_t^2$ is an exponential GARCH model, ω is a parameter of the ARCH model, β is the magnitude of the effect of positive issues on the current variety, γ is the magnitude of the effect of last period's volatility affecting the current variety and λ is parameter of GARCH model.

2) TGARCH Model

The *Threshold GARCH* (TGARCH) model is a development of the model (EGARCH) and the GJR-GARCH model. Given Y_t is the random variable iid (independent identical distribution) with E ($\underline{Y}_t$) = 0 and Var (Y_t) = 1. Then (e_t) is called the Threshold GARCH process (p, q) if it satisfies an equation of form,

$$\begin{cases} e_t = \sigma_t Y_t \\ \sigma_t = \theta_0 + \sum_{i=1}^{p} \theta_i^{(1)} e_{t-i}^{(1)} - \theta_i^{(2)} e_{t-i}^{(2)} + \sum_{j=1}^{q} \lambda_j \sigma_{t-j} \end{cases}$$

where $e_t^{(1)} = \max(e_t, 0)$, $e_t^{(2)} = \min(e_t, 0)$ dan $e_t = e_t^{(1)} - e_t^{(2)}$ are the effects of the threshold. The variables θ_0, $\theta_i^{(1)}$, $\theta_i^{(2)}$, and λ_i are native numbers (Francq and Zakoian, 2010). Based on the equation (2.25), the value of σ_t^2 is

$$\sigma_t^2 = \theta_0 + \sum_{i=1}^{p} \theta_i e_{t-i}^2 + \gamma_i e_{t-1}^2 d_{(e_{t-i})>0} + \sum_{j=1}^{q} \lambda_j \lambda_j^2.$$

Conditions in the event of good news ($\varepsilon_t > 0$) and bad news ($\varepsilon_t < 0$) give a different effect on the variety. The influence of good news is shown by θ while the influence of bad news is shown by ($\theta + \gamma$). If $\gamma \neq 0$, then there is an asymmetric effect. The e_t series has an average of zero and no correlation. Let y_t be the observational set during time t, with $t = 1, 2, ..., t$ being influenced by the exogenous variable x_t' where x_t' is the vector of the weak independent variable of size n_t, d is the parameter vector or coefficient of the exogenous variable. Parameters d, θ_0, θ_i, λ_j, and γ_i are parameters in the estimation, whereas γ_i is also a leverage effect.

3) APARCH Model

Ding *et al.* (1993) developed the (APARCH) model which is used to improve the weaknesses of ARCH and GARCH models in capturing the asymmetric power of good news and bad news in volatility. Bad news means that information will have a negative impact on the volatility, such as a drastic increase in fuel prices and a sharp rise in inflation. Good news means that information will have a positive impact on the volatility, such as a sharp increase in sales, decreased loan interest rates and business expansion. The general form of the APARCH model (p, q) is:

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i(|e_{t-1}| - \gamma_i e_{t-1})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta$$

and $\omega > 0$, $\delta > 0$, and $-1 < \gamma_i < 1$ and are estimates, δ estimated using Box Cox transform in standard deviation condition, $\gamma_i's$ are leverage effects. If the leverage effect is positive, meaning that bad news has stronger influence compared to good news, and vice versa, e_t is the t-th residual data.

E. Information Criteria

There are two criteria that can be considered in determining the best model, they are Akaike

Information Criterion (AIC) (Akaike, 1973; Akaike, 1974) with formula:

$$AIC_C = 2k - 2\ln(\hat{L})$$

and *Schwarz Criterion (SC)* (Schwarz, 1978) with formula:

$$SC = \ln(n)\, k - 2\ln(\hat{L})$$

where $\hat{L} = p(x|\hat{\theta}, M)$, $\hat{\theta}$ are the parameter values that maximize the likelihood function, $x = $ the observed data, n = the number of data points in x, and k = the number of parameters estimated by the model. Both criterions are used to select a model without test. A model is said to be interconnected from the second model if and only if the collection of independent variables of the first model is part of the independent variable of the second model. In practice the determination of a best model can be done by looking at the lowest values of AIC and SC.

III. MATERIALS AND METHODS

The data used in this paper is the daily stock price return data from Unilever Indonesia Tbk. during period of February 11, 2012 to November 10, 2017. To forecast the best asymmetric volatily models, first we identify the assumption of stationarity of the data graphically and use the Augmented Dickey Fuller (ADF) test. If the data meet the assumptions, the next step is to forecast the best ARMA(p,q) models that indicates the best Box-Jenkins models in certain lags using Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots. The next step is estimating the best ARMA(p,q) models parameters using Akaike Information (AIC) and Schwarz Criterions (SC) values. Afterwards, we test the ARCH effect using ARCH-LM and test the asymmetry in volatility using sign bias test before estimating EGARCH, TGARCH, and APARCH models. Finally, to determine the best asymmetric volatility model, we evaluate the smallest values of the AIC and SC values of the models.

IV. RESULTS AND DISCUSSIONS

A. Identification

Identification of the assumption of stationarity of the data graphically shows that daily stock price return data from Unilever Indonesia Tbk. is stationary either in the mean or variances. However, to ensure the stationary,

we do a unit root test (ADF-test) with hypothesis. The null hypothesis of a unit root is rejected (P-value = 0.0000) which means the return data is stationary. We therefore conclude that the time series are stationary at level and we can proceed to model ARMA(p,q).
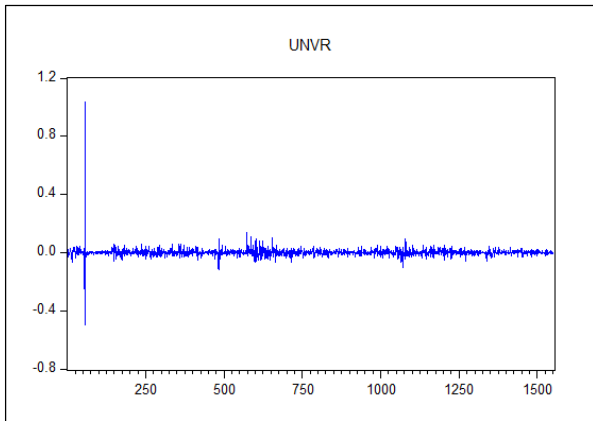

Figure 1. Return of Unilever Indonesia Tbk.Stock Price Data

B. Selection of ARMA(p,q)

To select the best ARMA(p,q), first we plot the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) as shown in Figure 2.
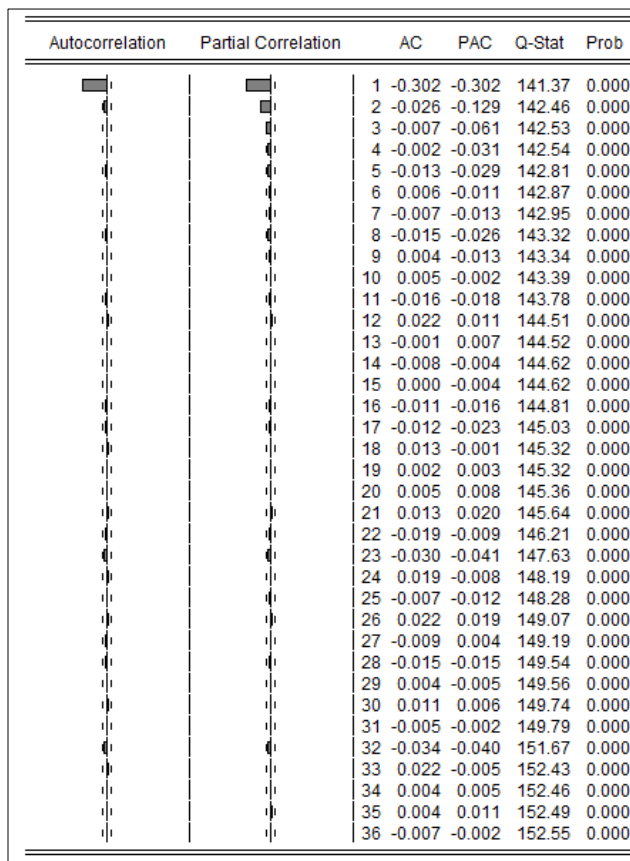

Figure 2. Correlogram return Unilever Indonesia Tbk.

The correlogram plot of ACF and PACF shows thatlags 1and 2 are significantly different from other lags. This indicates the best Box-Jenkins models most probably are in those lags. Referring tothe plot, evaluation ofARMA (1,0), (0,1), (1,1), ARMA (2,0), ARMA (0,2), and ARMA (2,2) models are carried out.

We use AIC and SC to select the best parameters p and q of ARMA to fit in the series. The result of the ARMA(p,q) selection models is shown in Table 1. The table shows that all parameters for ARIMA (0,1) are significant and has the lowest values of AIC and SC compared to other models. This indicate that the best suited model for the mean equation is an ARMA (0,1) model for all time series.To ensure the result, residual correlogram was used and show that ARMA(0,1) is really the best model among all the ARMA(p,q) models.

Table 1. Selection of ARMA(p,q)

No.	Model	Parameter	Parameter Estimate	P-Value	AIC	SC
1	ARMA (1,0)	β_1	-0.300011	0.0000	-3.92939	-3.92594
		β_1	-0.301622	0.0000		
2	ARMA (0,1)	α_1	-0.360420	0.0000	-3.94919	-3.94574
		α_1	-0.365618	0.0000		
5	ARMA (1,1)	β_1	0.106622	0.1232	-3.94896	-3.94206
		α_1	-0.454168	0.0000		
		α_1	-0.472038	0.0000		
7	ARMA (2,0)	β_1	-0.338130	0.0000	-3.94366	-3.93676
		β_2	-0.126629	0.0000		
		β_1	-0.340764	0.0000		
		β_2	-0.129243	0.0000		
9	ARMA (0,2)	α_1	-0.348563	0.0000	-3.94918	-3.94229
		α_2	-0.036189	0.1543		
		α_1	-0.352629	0.0000		
		α_2	-0.365618	0.1112		
11	ARMA (2,2)	β_1	0.907156	0.0070	-3.94622	-3.93242
		β_2	-0.058230	0.5323		
		α_1	-1.255655	0.002		
		α_2	0.342298	0.0607		

C. ARCH and GARCH Test

In the next step we test the ARCH effect using Lagrange Multiplier (LM) test. The result is presented in Table 2.

Table 2. ARCH-LM effect

Statistics/Probability	Values
F-statistic	295.8919
Obs*R-squared	248.7307
Prob. F(1,1192)	0.0000
Prob. Chi-Square(1)	0.0000

By looking at the probability of χ^2-statistic of ARCH-LM test (p-value = 0.0000), it can be concluded that the squared residuals from previous lags are correlated with the squared residual at time t. This indicates the existence of heteroscedasticity on the return data. As a result GARCH model can be used to the data. From the evaluation of GARCH models based on the AIC and SC values, the results shows that GARCH(1,0) model has all parameters significant and lowest values of AIC and SC. This indicate that GARCH(1,0) is better than others. Diagnostic checking for GARCH(1,0) model using Ljung Box-Pierce gives significant result with p-value > 0,5 which indicating the model is appropiate. Therefore, GARCH(1,0) model is good to make a better estimate for returns data. The GARCH(1,0) model for returns data of Unilever Indonesia Tbk. is:

$$\sigma_t^2 = -0.000830 + 0.171417\,\varepsilon_{t-1}^2$$

In the following step we evaluate the presence of volatility in the returns data using sign bias test. The analysis of sign bias test has p-value=0.0000. The null hypothesis of the test was rejected. It can be concluded that positive and negative shocks impact the volatility differently. Asymmetric GARCH models could therefore perform well in explaining conditional volatility for the return of Unilever Indonesia Tbk. stock price data. The usage of an asymmetric GARCH model is hence justified by the test.

D. Estimation and Comparison of Volatility Asymmetrics Models

Estimation of a series of asymmetric GARCH-family models to explain conditional variance and volatility clustering using Ljung-Box on various lags gives result of EGARCH(1,1), TGARCH(1,1) and APARCH(1,3) are best three models among all models in the lags. All parameters of EGARCH(1,1) having p-value < 0.01. Similar result for estimation of TGARCH model gives all parameters of TGARCH(1,1) having p-value < 0.01. Estimation of APARCH model gives the parameters of APARCH (1,3) having p-value < 0.01. This indicates that EGARCH(1,1), TGARCH(1,1), APARCH (1,3) models are appropiate for forecasting the stock price returns of Unilever Indonesia Tbk. period 2012-2017.

To compare the best performance of TGARCH(1,1), EGARCH(1,1) and APARCH (1,3) models, AIC and SC are used. The summary of the comparison performance of the three models is presented in Table 3. Based on the values of AIC and SC, it can be concluded that APARCH(1,3) model outperfoms the other models since it has statistically significant estimation of all parameters and smallest AIC and SC values.

Table 3. Comparison of performance asymmetric volatility model

No	Model	Parameter	Coefficient	P-value	AIC	SC
1	EGARCH (1,1)	ω	-11.6172	0.0000	-4.67259	-4.65536
		θ_1	0.09259	0.0000		
		γ_1	-0.23949	0.0000		
		λ_1	-0.54011	0.0000		
2	TGARCH (1,1)	ω	5.14E-5	0.0000	--4.78279	-4.76554
		θ_1	0.15533	0.0000		
		γ_1	0.52850	0.0000		
		λ_1	0.67815	0.0000		
3	APARCH (1,3)	ω	0.00067	0.0000	-4.84165	-4.81407
		α_1	0.23477	0.0000		
		γ_1	0.30894	0.0000		
		β_1	1.43225	0.0000		
		β_2	-0.77521	0.0000		
		β_3	0.14983	0.0000		
		δ	1.20962	0.0000		

IV. CONCLUSION

Taking the 2012-2017 period of sampel and using daily observations from Unilever Indonesia Tbk. stock price returns data, we find that volatility does exist in the data. The asymmetric volatility APARCH(1,3) model is the best suited specifications model than EGARH(1,1) and TGARCH(1,1) models corresponding to the data. The empirical performance of the asymmetric volatility, APARCH(1,3) model has AIC and SC scores are substantially lowerand has all statistically significant estimation parameters probability < 0.05. The forecasting APARCH (1,3) for stock price returns data of Unilever IndonesiaTbk. is the following formula:

$$\sigma_t^{1.209624} = 0.000675 + 0.234766(|\varepsilon_{t-1}| - 0.308938\varepsilon_{t-1})^{1.209624} + 1.432246(\sigma_{t-1})^{1.209624} - 0.775210(\sigma_{t-2})^{1.209624} + 0.149835(\sigma_{t-3})^{1.209624}.$$

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