GENERALIZED METHOD OF MOMENTS' CHARACTERISTICS AND ITS APPLICATION ON PANELDATA

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ABSTRACT: Generalized Method of Moments (GMM) is an estimation procedure that allows econometric models especially in panel data to be specified while avoiding often unwanted or unnecessary assumptions, such as specifying a particular distribution for the errors. Panel data is combination of time series and cross section data that contain observations on thousands of individuals or families, each observed at several points in time. Furthermore, the Generalized Method of Moments estimator is obtained by minimizing the criterion function by making sample moment match the population moment.The point of this research is to analyze characteristics GMM estimator on panel data fixed effect models especially unbiasedness, variance minimum, consistency, and normal asymptotic distributed estimator properties. This paper also provide the application of GMM estimation on the area of "Cost for United States Airlines on Six Firms from 1970-1984".

Keywords: Panel Data; Generalized Method of Moments; Unbiasedness; Variance Minimum; Consistency; Normal Asymptotic Distributed.

1. INTRODUCTION

Econometrics is the field of economics that concerns itself with the application of mathematical statistics and the tools of statistical inference to the empirical measurement of relationships postulated by economic theory (Greene, 2008)[1].The methodologies that combine mathematical statistics and economics theory produce an econometrics model. Many recent studies in econometrics model have analyzed panel or longitudinal data sets that combine time series and cross section data sets (Johnston, 1984)[2].

Panel data sets analyzed time series data on sets of firms, states, countries, or industries simultaneously so its model linear may be written as follows:

$$y_{it} = x_{it}'\,\beta + z_i'\alpha + \varepsilon_{it} \qquad (1)$$

there are *K* parameter slope in x_{it}' , with $i = 1,2,\ldots, N$ show analysis in cross section and $t = 1,2,\ldots,T$ show analysis in time series. The vector $z_i'\alpha$ is called individual effect with z_i' contains a constant term and a set of individual or group specific variables (Greene, 2008)[3]. The various cases of individual effect on panel data are pooled regression, fixed effect, and random effect.

Gujarati (2004)[4] wrote that using panel data giving more data and information so increasing degree of freedom, anticipating heteroscedaticity problem and provide better estimation econometrics. On panel data analysis, often produces over determined systems where there are more moment equations than number of parameters. Hansen (1982)[5] introduced the estimation method to solve this case is Generalized Method of Moments (GMM) by minimizing criterion weighted function. Generalized Method of Moments is convenient for estimating interesting extensions of the basic unobserved effect model (Wooldridge, 2001)[6].

The purpose of this paper is to prove characteristics of GMM estimator on panel data especially unbiasedness, variance minimum, consistency, and normal asymptotic distributed properties. To show all of the properties, Section 2 will presents parameter estimation on panel data fixed effect model linear using GMM. Furthermore, Section 3 will show unbiasedness estimator property, Section 4 will show variance minimum property, consistency property will be shown in Section 5 and Section 6 will discuss asymptotic normal distributed. Finally, Section 7 will present estimation of GMM estimator to estimate linear model panel data sets "*Cost for United States Airlines on 6 Firms from 1970-1984*".

2. PARAMETER LINEAR MODEL PANEL DATA ESTIMATION USING GMM

In exactly identified cases, where number of equation moments equals to number of parameters there will be a single solution by Method of Moments. But, when the number of moment conditions exceeds the number of parameters, we cannot hope to obtain an estimator by setting the empirical equivalent $\bar{g}(\theta)$ of our moment condition equal to zero, (de Jong and Han, 2000)[7]. In other word, over determined system there is no unique solution so it will be necessary to minimize criterion function as the criterion a weighted sum of squares

$$q = \overline{m}(\beta)'W\,\overline{m}(\beta),$$

this estimation method is called Generalized Method of Moment (GMM).

The linear model panel data fixed effect is written as:

$$y_{it} = x_{it}'\,\beta + \alpha_i + \varepsilon_{it}$$

where β is *K x 1* parameter vector and $\alpha_i = z_i\,\alpha$, embodies all the observable effects and specifies an estimable conditional mean. This fixed effects approach takes α_i to be a group-specific constant term in the regression model (Greene, 2008).

The preceding linear model in Section 1 help us to make sample moments equation as below:

$$\left[\frac{1}{n}\sum_{i=1}^{n} \boldsymbol{z}_i\left(y_i - \boldsymbol{x}_i'\boldsymbol{\beta}\right)\right] = \left[\frac{1}{n}\sum_{i=1}^{n} \boldsymbol{m}_i(\hat{\boldsymbol{\beta}})\right] = \bar{\boldsymbol{m}}(\hat{\boldsymbol{\beta}}) = \boldsymbol{0}$$

Using GMM, the criterion a weighted sum of squares is defined as

$$Min_{\beta}\, q = \bar{\boldsymbol{m}}(\hat{\boldsymbol{\beta}})'\boldsymbol{W}\bar{\boldsymbol{m}}(\hat{\boldsymbol{\beta}})$$

then minimizing *q* as follows:

$$\frac{\partial q}{\partial \beta} = \boldsymbol{0}$$

$$\frac{\partial \bar{m}(\hat{\beta})' W \bar{m}(\hat{\beta})}{\partial \beta} = 0$$

$$2\frac{\partial \overline{m}(\hat{\beta})'}{\partial \beta} W \,\overline{m}(\hat{\beta}) = 0 . \qquad (2)$$

From exactly identified, we get the solution of easily as:

$$\overline{m}(\hat{\beta}) = \left(\frac{1}{n} Z'y\right) - \left(\frac{1}{n} Z'x\right)\hat{\beta} \qquad (3)$$

Then substitute (2) to (3) we will get:

$$2\left(\frac{\partial}{\partial \beta}\left[\left(\frac{1}{n}Z'y\right) - \left(\frac{1}{n}Z'X\right)\hat{\beta}\right]\right) W \left[\left(\frac{1}{n}Z'y\right) - \left(\frac{1}{n}Z'X\right)\hat{\beta}\right] = 0$$

$$2\left(-\frac{1}{n}Z'X\right)' W \left[\left(\frac{1}{n}Z'y - \frac{1}{n}Z'X\hat{\beta}\right)\right] = 0$$

$$-\frac{2}{n^2}\left[(X'Z)W(Z'y - Z'X\hat{\beta})\right] = 0$$

$$\left[(X'Z)W(Z'y - Z'X\hat{\beta})\right] = 0$$

Associative property on matrix algebra allows that:

$$(X'Z)W(Z'y) - (X'Z)W(Z'X\hat{\beta}) = 0$$

So we get

$$\hat{\beta} = [(X'Z)W(Z'X)]^{-1}(X'Z)W(Z'y) .$$

In over identified case ($L > K$), the weighted matrix $\boldsymbol{W}$ can be identity $\boldsymbol{I}$ or inverse of covariance matrix V^{-1}. Furthermore analysis show that efficient and consistent estimator is obtained by using inverse of asymptotic covariance V^{-1}, with

$$V_{GMM} = \frac{1}{n}[G'\Phi^{-1}G]^{-1}$$

where

$$\Phi = Var[\sqrt{n}(\bar{m} - \beta)]$$

and

$$G = \frac{\partial \overline{m}_l(\hat{\beta})'}{\partial \beta'} .$$

So, the GMM estimator on panel data fixed effect model can be written as

$$\hat{\beta} = \left[(X'Z)\hat{V}^{-1}(Z'X)\right]^{-1}(X'Z)\hat{V}.$$

3. UNBIASEDNESS PROPERTY OF GMM ESTIMATOR

From the result of parameter estimation using GMM in Section 2, then the estimator $\hat{\beta}_{GMM}$ can be rewritten as

$$\hat{\beta}_{GMM} = My$$

where

$$M = [(X'Z)\hat{V}^{-1}(Z'X)]^{-1}(X'Z)\hat{V}^{-1}Z' .$$

Thus

$$E(\hat{\beta}_{GMM}) = E(My)$$

$$= ME(y)$$

$$= \left\{\left[(X'Z)\hat{V}^{-1}(Z'X)\right]^{-1}(X'Z)\hat{V}^{-1}Z'\right\} \cdot E(y)$$

$$= \left\{\left[(X'Z)\hat{V}^{-1}(Z'X)\right]^{-1}(X'Z)\hat{V}^{-1}Z'\right\} \cdot E(X\beta + \varepsilon)$$

Since $X\beta$ is not random variable and $E(\varepsilon) = 0$, we get

$$= \left\{\left[(X'Z)\hat{V}^{-1}(Z'X)\right]^{-1}(X'Z)\hat{V}^{-1}Z'\right\} \cdot X\beta$$

$$= (Z'X)^{-1}\hat{V}[(X'Z)^{-1}(X'Z)]\hat{V}^{-1}Z'X\beta$$

$$= (Z'X)^{-1}[\hat{V}\hat{V}^{-1}]Z'X\beta$$

$$= X^{-1}[(Z')^{-1}Z']X\beta$$

$$= [X^{-1}X]\beta$$

$$= \beta$$

So, it is proven that $\hat{\beta}_{GMM}$ is unbiased estimator of β.

4. VARIANCE MINIMUM PROPERTY OF GMM ESTIMATOR

Econometrics model estimations using GMM is one of semiparametric estimation types that move away from parametric assumptions, such as specifying a particular distribution for the errors. Sometimes it makes some difficultness to analyze characteristic of an estimator. But, the semiparametric efficiency bound is associated with the minimum variance that plays the role of the Fisher Information bound in a semiparametric setting as mentioned by Nekipelov (2010)[8]. Since GMM estimator has normal asymptotic normal distributed property where its probability density function is form of exponential family.

Hogg and Craig (1995)[9] defined that exponential class one parameter has probability density function of the continuous type as follows:

$$f(x;\theta) = \begin{cases} \exp\{p(\theta)K(x) + S(x) + q(\theta)\}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

where:

1. Neither a nor b depends upon $\theta, \gamma < \theta < \delta$,
2. $p(\theta)$ is a nontrivial continuous function of $\theta, \gamma < \theta < \delta$,
3. Each of $K'(x) \neq 0$ and $S(x)$ is a continuous function of x, $a < x < b$

Since asymptotic property, it can be assumed that disturbances have normal multivariate distribution with mean θ and matrix covariance V, $\varepsilon \sim N(\theta, V)$ as

$$f(X;\theta) = \frac{1}{\sqrt{2\pi|V|}} exp\left[\frac{-(X-\theta)'V^{-1}(X-\theta)}{2}\right]$$

Thus

$$\ln f(X;\theta) = \ln\left\{\frac{1}{\sqrt{2\pi|V|}} exp\left[\frac{-(X-\theta)'V^{-1}(X-\theta)}{2}\right]\right\}$$

$$= -\frac{1}{2}\ln(2\pi|V|) - \frac{(X-\theta)'V^{-1}(X-\theta)}{2}$$

The derivative of $\ln f(X;\theta)$ with respect to θ as follows:

$$\frac{\partial \ln f(X;\theta)}{\partial\theta} =$$

$$= \frac{\partial}{\partial\theta}\left[-\frac{1}{2}\ln(2\pi|V|) - \frac{(X-\theta)'V^{-1}(X-\theta)}{2}\right]$$

$$= \frac{\partial}{\partial\theta}\left[-\frac{(X-\theta)'V^{-1}(X-\theta)}{2}\right]$$

$$= -\frac{1}{2}\left\{\frac{\partial}{\partial\theta}[(X'V^{-1} - \theta'V^{-1})(X-\theta)]\right\}$$

$$= -\frac{1}{2}\left\{\frac{\partial}{\partial\theta}[X'V^{-1}X - X'V^{-1}\theta - \theta'V^{-1}X + \theta'V^{-1}\theta]\right\}$$

$$= -\frac{1}{2}[0 - X'V^{-1} - V^{-1}X + 2V^{-1}\theta]$$

$$= -\frac{1}{2}[-2V^{-1}X + 2V^{-1}\theta]$$

$$= V^{-1}(X-\theta)$$

And the second derivative $\ln f(X;\theta)$ with respect to θ is:

$$\frac{\partial^2 \ln f(X;\theta)}{\partial\theta^2} = \frac{\partial}{\partial\theta}[V^{-1}(X-\theta)] = V^{-1}$$

We get Fisher Information as:

$$I(\theta) = \left(E\left[\frac{\partial \ln f(X;\theta)}{\partial\theta}\right]\right)^2 =$$

$$-E\left[\frac{\partial^2 \ln f(X;\theta)}{\partial\theta^2}\right] = -E[-V^{-1}] = V^{-1}$$

So the Rao-Cramer lower bound is:

$$Var(T) \geq \frac{\left[\frac{\partial}{\partial\theta}g(\theta)\right]^2}{n\cdot I(\theta)}$$

$$Var(T) \geq \frac{\begin{pmatrix} \frac{\partial x_{1i}}{\partial\theta_1} & \frac{\partial x_{1i}}{\partial\theta_2} & \cdots & \frac{\partial x_{1i}}{\partial\theta_n} \\ \frac{\partial x_{2i}}{\partial\theta_1} & \frac{\partial x_{2i}}{\partial\theta_2} & \cdots & \frac{\partial x_{2i}}{\partial\theta_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial x_{ni}}{\partial\theta_1} & \frac{\partial x_{ni}}{\partial\theta_2} & \cdots & \frac{\partial x_{ni}}{\partial\theta_n} \end{pmatrix}'\begin{pmatrix} \frac{\partial x_{1i}}{\partial\theta_1} & \frac{\partial x_{1i}}{\partial\theta_2} & \cdots & \frac{\partial x_{1i}}{\partial\theta_n} \\ \frac{\partial x_{2i}}{\partial\theta_1} & \frac{\partial x_{2i}}{\partial\theta_2} & \cdots & \frac{\partial x_{2i}}{\partial\theta_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial x_{ni}}{\partial\theta_1} & \frac{\partial x_{ni}}{\partial\theta_2} & \cdots & \frac{\partial x_{ni}}{\partial\theta_n} \end{pmatrix}}{n\cdot V^{-1}}$$

$$\geq \frac{\mathcal{J}'\mathcal{J}}{n\cdot V^{-1}}$$

$$\geq \frac{1}{n}[\mathcal{J}'V\mathcal{J}]$$

As we have defined earlier in section 2, that

$$V_{GMM} = \frac{1}{n}[G'\Phi^{-1}G]^{-1}$$

where

$$\Phi = Var[\sqrt{n}(\bar{m} - \beta)]$$

and

$$G = \frac{\partial \bar{m}_l(\hat{\beta})'}{\partial\beta'}.$$

So we can write variance of Rao-Cramer and variance of GMM estimator as relationship as follows:

$$\frac{1}{n}[\mathcal{J}'V\mathcal{J}] > \frac{1}{n}[G'\Phi^{-1}G]^{-1}$$

$$[\mathcal{J}'V\mathcal{J}] > [G'\Phi^{-1}G]^{-1}$$

Since variance of GMM estimator less than Rao-Cramer lower bound then it is proven that variance of GMM estimator has variance minimum.

5. CONSISTENCY PROPERTY OF GMM ESTIMATOR

We have discussed that GMM estimator is obtained by minimizing criterion function

$$q_n(\beta) = \bar{m}_n(\beta)'W_n\bar{m}_n(\beta)$$

where

$$\bar{m}_n(\beta) = \frac{1}{n}\sum_{i=1}^{n} z_i(y_i - x_i'\beta)$$

And W_n is positive definite matrix as discussed in Newey (1985).

It must first be established that $q_n(\beta)$ converges to a value $q_0(\beta)$, where

$$\lim_{n\to\infty} q_0(\beta) = \lim_{n\to\infty} \bar{m}_n(\beta)'W_n\bar{m}_n(\beta)$$

$$= \lim_{n\to\infty}\left\{\left[\frac{1}{n}\sum_{i=1}^{n} z_i(y_i - x_i'\beta)\right]W_n\left[\frac{1}{n}\sum_{i=1}^{n} z_i(y_i - x_i'\beta)\right]\right\}$$

$$= \lim_{n\to\infty}\left\{\frac{1}{n^2}\left(\sum_{i=1}^{n} z_i(y_i - x_i'\beta)\right)^2 W_n\right\} = 0$$

So, $q_n(\beta)$ converges to 0.

For the proof of $p\lim \hat{\beta}_{GMM} = \beta$ reader see Greene (2008)[10]. So, GMM estimator is consistent estimator.

6. NORMAL ASYMPTOTICALLY DISTRIBUTED PROPERTY OF GMM ESTIMATOR

Asymptotic normality of GMM estimators follows from taking a mean value expansion of the moment conditions around the true parameter, see (Chen et al, 2002). To show normal asymptotic distributed property, the first order condition for the GMM estimator are:

$$\frac{\partial q_n(\hat{\beta}_{GMM})}{\partial \hat{\beta}_{GMM}} = 0$$

$$\frac{\partial}{\partial \hat{\beta}_{GMM}}\left(\bar{m}_n(\hat{\beta}_{GMM})' W_n \bar{m}_n(\hat{\beta}_{GMM})\right) = 0$$

$$2\,\frac{\partial \bar{m}_n(\hat{\beta}_{GMM})'}{\partial \hat{\beta}_{GMM}} W_n \bar{m}_n(\hat{\beta}_{GMM}) = 0$$

Let $\bar{G}_n(\hat{\beta}_{GMM}) = \frac{\partial}{\partial \hat{\beta}_{GMM}} \bar{m}_n(\hat{\beta}_{GMM})$.

Then

$$2\bar{G}_n(\hat{\beta}_{GMM})' W_n \bar{m}_n(\hat{\beta}_{GMM}) = 0$$

$$\bar{G}_n(\hat{\beta}_{GMM}) W_n\, \bar{m}_n(\hat{\beta}_{GMM}) = 0 \qquad (4)$$

The orthogonality equations (4) are assumed that vector $\bar{m}_n$ to be continuous at closure interval $[\beta_0, \hat{\beta}_{GMM}]$ and continuously differentiable at $(\beta_0, \hat{\beta}_{GMM})$ so there are $\bar{\beta} \in (\beta_0, \hat{\beta}_{GMM})$, and this allows us to employ the Mean Value Theorem

$$\frac{\partial \bar{m}_n(\bar{\beta})}{\partial \hat{\beta}_{GMM}} = \frac{\bar{m}_n(\hat{\beta}_{GMM}) - \bar{m}_n(\beta_0)}{\hat{\beta}_{GMM} - \beta_0}$$

or it can be written as

$$\bar{m}_n(\hat{\beta}_{GMM}) = \bar{m}_n(\beta_0) + \frac{\partial \bar{m}_n(\bar{\beta})}{\partial \hat{\beta}_{GMM}}(\hat{\beta}_{GMM} - \beta_0)$$

$$\bar{m}_n(\hat{\beta}_{GMM}) = \bar{m}_n(\hat{\beta}_0) + \bar{G}_n(\bar{\beta})(\hat{\beta}_{GMM} - \hat{\beta}_0) \qquad (5)$$

where $\bar{\beta}$ is a point between $\hat{\beta}_{GMM}$ and the true parameter β_0.

Substitute (4) to the (5) and we get

$$\bar{G}_n(\hat{\beta}_{GMM})' W_n[\bar{m}_n(\beta_0) + \bar{G}_n(\bar{\beta})(\hat{\beta}_{GMM} - \beta_0)] = 0$$

$$\bar{G}_n(\hat{\beta}_{GMM})' W_n \bar{m}_n(\beta_0) + \bar{G}_n(\hat{\beta}_{GMM})' W_n \bar{G}_n(\bar{\beta})(\hat{\beta}_{GMM} - \beta_0) = 0$$

$$\bar{G}_n(\hat{\beta}_{GMM})' W_n \bar{G}_n(\bar{\beta})(\hat{\beta}_{GMM} - \beta_0) = -\bar{G}_n(\hat{\beta}_{GMM})' W_n \bar{m}_n(\beta_0)$$

Using left cancelation law by $[\bar{G}_n(\hat{\beta}_{GMM}) W_n \bar{G}_n(\bar{\beta})]^{-1}$, obtained $(\hat{\beta}_{GMM} - \beta_0) =$

$$-[\bar{G}_n(\hat{\beta}_{GMM})' W_n \bar{G}_n(\bar{\beta})]^{-1} \bar{G}_n(\hat{\beta}_{GMM})' W_n \bar{m}_n(\beta_0)$$

And multiply by $\sqrt{n}$, produces

$$\sqrt{n}(\hat{\beta}_{GMM} - \beta_0) =$$

$$-[\bar{G}_n(\hat{\beta}_{GMM})' W_n \bar{G}_n(\bar{\beta})]^{-1} \bar{G}_n(\hat{\beta}_{GMM})' W_n \sqrt{n}\bar{m}_n(\beta_0)$$

now the quantities on the left- and right-hand sides have the same limiting distribution that is $N[(\beta, V_{GMM})$. Furthermore see Greene (2008), and we have asymptotic normal distribution with mean β and variance V_{GMM} ,

$$\hat{\beta}_{GMM} \sim N[\beta, V_{GMM}]$$

7. APPLICATION OF GMM ESTIMATION ON PANEL DATA SETS

In this section, we will presents numeric analysis on panel data sets of "Cost for United States Airlines on Six Firms from 1970-1984 (15 years)" by http://www.indiana.edu/=statmath/stat/all/panel/airline.dta has been accessed on 20th December 2013. Using program R3.0.1, we get the panel data linear model about cost for United States airlines on six firms from 1970-1984 with GMM is $\hat{Y} = 0.7708 * X_1 + 1.0743 * X_2 + 0.9654 * X_3$ and shown in the table as follows:

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Table1: Parameter Estimation Using GMM

Method	Estimator	Estimation	Standard Error Mean
GMM	$\hat{\beta}_1$	0.7708356	0.0934
	$\hat{\beta}_2$	1.0743491	0.0919
	$\hat{\beta}_3$	0.9653712	0.0053

From the table, the estimations of the distribution variance of sample mean are 0.0934, 0.0919 and 0.0053. The measure of standard error is influenced by standard deviation of population and number of sample. Actually we should have expected the GMM estimator to improve the standard errors. As a comparison, will be presents parameter estimation using Feasible Generalized Least Square (FGLS) method is presented as below:

Table2: Parameter Estimation Using FGLS

Method	Estimator	Estimation	Standard Error Mean
FGLS	$\hat{\beta}_1$	0.89784	0.01459
	$\hat{\beta}_2$	1.19594	0.04607
	$\hat{\beta}_3$	-2.03970	0.46191

So, we have panel data linear model about cost for United States airlines on six firms from 1970-1984 using FGLS $\hat{Y} = 0.8978 * X_1 + 1.1959 * X_2 - 2.03970 * X_3$.

From the table we can say that estimation for β_1 is 0.89784 with standard error mean 0.01459. While the estimation for β_2 is 1.19594 with standard error mean 0.04607 and for β_3 is -2.03970 with standard error mean 0.46191. In fact, the standard error of mean of and by using GMM is bigger than using FGLS but

standard error of mean of

by using GMM is smaller than using FGLS. And the graph of Y and using GMM is shown as follows:

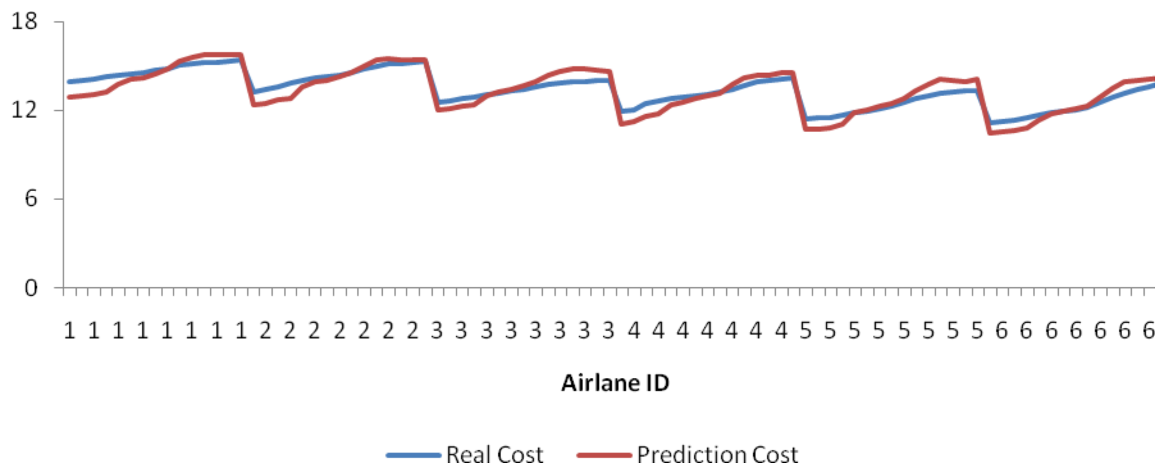

Figure 1: Plot Y and Cost of Six Airlines Using GMM

The figure represent total cost of six airlines for fifteen years so that we have 90 (ninety) number of cases. Blue line shows the real value of total costs on six firms airlines from 1970-1984 and red line states the estimation of total costs on six firms airlines from 1970-1984. For example, the first airline in 1970, we have total cost of 13.94710 but using the estimation GMM we have 12.91675. From the figure we can state that estimation of total cost has closed value to real total cost. In the center of every hills, the estimation is similar with real value.

8. CONCLUSION

For the general case of the instrumental variable estimator, there are exactly as many moment equations as there are parameters to be estimated. Thus, each of these are exactly identified cases. There will be a single solution to the moment equations, this is called Method of Moment Estimation. But there are cases in which there are more moment equations than parameters, so the system is over determined. The Generalized Method of Moments technique is an extension of the Method of Moments by minimizing criterion function as the criterion a weighted sum of squares. In fact, a large proportion of the recent empirical work in econometrics, particularly in macroeconomics and finance, has employed GMM estimators. Based on the explanation in the previous chapters, we have that Generalized Method of Moments estimator on panel data linear model has characteristics as unbiasedness, variance minimum, consistency and normally asymptotic distributed property. From numeric analysis on panel data sets of "Cost for United States Airlines on Six Firms from 1970-1984 (15 years)" using GMM we have model linear as $\hat{Y} = 0.7708 * X_1 + 1.0743 * X_2 + 0.9654 * X_3$. The measure of standard error is influenced by standard deviation of population and number of sample. Actually we should have expected the GMM estimator to improve the standard errors. From the figure plot Y and , we can state that estimation using GMM of total cost has closed value to real total cost, Although, we realize that estimation using GMM on real panel data sometimes will be biased.

REFERENCES

Chen, X., Linton, O. and Keilegom. I,V. 2002 *Estimation of Semiparametric Models when The Criterion Function is not Smooth.* CEMMAP working paper.

De Jong, R. and Chirok, H. 2000. *The Properties of L_p-GMM Estimators*. Michigan State University, USA.

Greene, W.H. 2008. *Econometric Analysis*.PrenticeHall, USA.

Gujarati, D.N. 2004. *Basic Econometrics*. McGraw Hill, USA.

Hansen, P.L. 1982. Large Sample Properties of Generalized Method of Moments Estimators, *Econometrica* ,Vol.20: 1029-1054.

Hogg, R.V. and Craig,A.T.1995. *Introduction to Mathematical Statistics*. Prentice-Hall, New Jersey.

Johnston, J. 1984. *Econometric Methods*. McGraw Hill, USA.

Nekipelov, D. 2010. *A Note on The Role of Regularity Conditions in a Class of Models with Inverse Weighting*. University of California, USA.

Newey, W. 1985. Generalized Method of Moments Specification Testing, *Journal of Econometrics* Vol.29: 229-256.

Wooldridge, J.M. 2001. *Application of Generalized Method of Moments Estimation*, *Journal of Economic Perspectives:* 87-100.

Appendix 1. Y Value and The Estimation Using GMM

Airlines	Year			Airlines	Year		
1	1970	13.94710	12.91675	4	1970	11.88564	11.04837
1	1971	14.01082	12.97791	4	1971	12.04468	11.19323
1	1972	14.08521	13.07349	4	1972	12.41919	11.54639
1	1973	14.22863	13.22924	4	1973	12.64236	11.72778
1	1974	14.33236	13.78041	4	1974	12.77801	12.32571
1	1975	14.41640	14.09737	4	1975	12.83185	12.53035
1	1976	14.52004	14.18012	4	1976	12.95019	12.77574
1	1977	14.65482	14.43735	4	1977	13.06900	12.96246
1	1978	14.78597	14.79953	4	1978	13.18551	13.14863
1	1979	14.99343	15.35413	4	1979	13.42509	13.72605
1	1980	15.14728	15.61570	4	1980	13.68818	14.21035
1	1981	15.16818	15.75955	4	1981	13.86622	14.34572
1	1982	15.20081	15.77612	4	1982	13.99255	14.34383
1	1983	15.27014	15.77030	4	1983	14.08048	14.51689
1	1984	15.37330	15.74729	4	1984	14.17805	14.54340
2	1970	13.25215	12.37964	5	1970	11.42257	10.68275
2	1971	13.37018	12.45999	5	1971	11.46613	10.70741
2	1972	13.56404	12.71241	5	1972	11.49463	10.81105
2	1973	13.81480	12.83174	5	1973	11.66106	11.07242
2	1974	14.00113	13.60834	5	1974	11.83777	11.81599
2	1975	14.12160	13.89774	5	1975	11.95907	11.99659
2	1976	14.22188	14.04386	5	1976	12.11816	12.25827
2	1977	14.35158	14.26279	5	1977	12.25587	12.49411
2	1978	14.52128	14.53040	5	1978	12.52097	12.82635
2	1979	14.75096	15.02174	5	1979	12.78525	13.35413
2	1980	14.95901	15.45870	5	1980	12.97698	13.73985
2	1981	15.08463	15.53138	5	1981	13.16981	14.07458
2	1982	15.12863	15.46155	5	1982	13.18237	14.02826
2	1983	15.19235	15.44143	5	1983	13.27328	13.97698
2	1984	15.25283	15.39542	5	1984	13.32164	14.08373
3	1970	12.56479	12.02883	6	1970	11.14154	10.44438
3	1971	12.64203	12.09713	6	1971	11.22396	10.53358
3	1972	12.74273	12.23954	6	1972	11.33653	10.61042
3	1973	12.83360	12.35569	6	1973	11.49423	10.76885
3	1974	13.01709	12.99271	6	1974	11.68224	11.35805
3	1975	13.14297	13.28480	6	1975	11.79931	11.78324
3	1976	13.26273	13.44731	6	1976	11.88492	11.91468
3	1977	13.41403	13.66847	6	1977	12.04773	12.10480
3	1978	13.57191	13.95441	6	1978	12.20495	12.28832
3	1979	13.72546	14.34397	6	1979	12.53104	12.93120
3	1980	13.85619	14.63267	6	1980	12.85181	13.51567
3	1981	13.93400	14.81400	6	1981	13.13620	13.97539
3	1982	13.90724	14.77627	6	1982	13.35884	14.00739
3	1983	13.99694	14.71668	6	1983	13.59784	14.11097
3	1984	13.97292	14.67299	6	1984	13.82497	14.21853