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ANALYSIS AND RATIO OF LINEAR FUNCTION OF PARAMETERS IN FIXED EFFECT THREE LEVEL NESTED DESIGN

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ABSTRACT

The aims of this study are first to build the linear model of the fixed effect three level nested design. The model is nonfull column rank and has a constraint on its parameters; second is to transform the nonfull column rank model with a constraint into full column rank and unconstraint model by using method of model reduction; and third is to derive statistics for testing various hypothesesby using Generalized Likelihood Ratio (GLR) test and to derive the ratio of linear function of parameters by using Fieller's Theorem. Based on the full column rank and unconstraint model the analysis to be conducted is: to estimate the parameters, to derive statistics for testing various hypotheses and to derive confidence intervals of the ratio of the linear function of parameters. The estimation of parameters and the statistics for testing some hypotheses are unbiased. Based on the simulation results, it can be shown that the tests are unbiased and in line with the criteria given by Pearson and Please. The simulation results for the (1-α) confidence interval of the ratio of the linear function of parameters tau (τ_i), beta ($\beta_{j(i)}$) and gamma ($\gamma_{k(ij)}$) are presented for different values of ρ's and in all cases the values of ρ's are contained in the 95% confidence intervals.

Keywords: nonfull rank model, full rank model, model reduction, estimation, testing hypotheses, ratio, linear function, parameters.

1. INTRODUCTION

In general linear model $Y = X\theta + \varepsilon$, sometimes the design matrix X is not full column rank. This condition implies that the estimation of parameter θ is not unique. There are some available methods to deal with this condition when the design matrix X is not full column rank. Among others the methods are mean model approach [1, 2], reparameterization approach [3, 4], and model reduction method[5] are used. Mustofa et al [6] in their study has discussed the transformation from constrained model into unconstrained model in two way treatment structure with interaction by using model reduction method. Mustofa *et al* [7] in their study discussed the combination of randomized complete block design (RCBDs) by using model reduction method. Mustofa *et al*[8] in their study have discussed the application of model reduction method to deal with the ratio of linear function of parameters in combination of two split plot designs.

In this study the authors would like to discuss the application of model reduction method [5] in fixed effect three level nested design, First, it will transform the constrained model into unconstrained model, and then will be discussed the estimation of parameters, testing hypotheses, and ratio of linear function of parameters in the unconstrained model by using Fieller's Theorem [9, 10].

2. MODEL REDUCTION METHOD

The linear model of the fixed effect tree level nested design is given below:

$$y_{ijkl} = \mu + \tau_i + \beta_{j(i)} + \gamma_{k(ij)} + \varepsilon_{ijkl} \begin{cases} i = 1,2,\dots a \\ j = 1,2,\dots b \\ k = 1,2,\dots c \\ l = 1,2,\dots n \end{cases} \quad (1)$$

where y_{ijkl} is the i-th observation from factor A, the j-th observation from factor B, the k-th observation from factor C and the l-th replication, μ is grand mean, τ_i is the i-th effect of factor A, $\beta_{j(i)}$ is the effect of the j-th factor B nested into i-th factor A,$\gamma_{k(ij)}$ is the k-th effect of factor C nested within i-th factor A and j-th factor B, and ε_{ijkl} is the error and ε_{ijkl} has a distribution $N(0,\sigma^2)$ [11, 12].

For the fixed effect model it is assumed that the model (1) has a restriction as follows:

$$\sum_{i=1}^{a}\tau_i = 0; \quad \sum_{j=1}^{b}\beta_{j(i)} = 0 \ \forall i; \quad \sum_{i=1}^{a}\beta_{j(i)} = 0 \ \forall j; \quad \sum_{k=1}^{c}\gamma_{k(ij)} = 0 \ \forall i,j; \quad \sum_{i=1}^{a}\gamma_{k(ij)} = 0 \ \forall j,k; \quad \sum_{j=1}^{b}\gamma_{k(ij)} = 0 \ \forall i,k \quad (2)$$

In the matrix form (1) and (2) can be written as follows:

$$Y = X\theta + \varepsilon \quad (3)$$

Constraint $G\theta = 0$
where

$$X = [1_{abcn} \quad I_a \otimes 1_{bcn} \quad I_{ab} \otimes 1_{cn} \quad I_{abc} \otimes 1_n]$$

$$\theta = [\mu, \tau_1, \tau_2.., \tau_a, \beta_{1(1)}, \beta_{2(1)} \dots, \beta_{b(1)}, \beta_{1(2)}, \beta_{2(2)}.., \beta_{b(2)}, .., \beta_{1(a)}, \beta_{1(a)}, \beta_{2(a)}, .., \beta_{b(a)}, \gamma_{1(11)}, \gamma_{2(11)} \dots, \gamma_{c(11)}, \gamma_{1(12)}, \gamma_{2(12)} \dots, \gamma_{2(12)} \dots, \gamma_{c(12)}, \dots, \gamma_{1(1b)}, \gamma_{2(1b)} \dots, \gamma_{c(1b)}, \gamma_{1(21)}, \gamma_{2(22)} \dots, \gamma_{c(22)}, \dots, \gamma_{1(2b)}, \gamma_{2(2b)}, \dots, \gamma_{c(2b)}, \dots \gamma_{1(a1)}, \gamma_{2(a2)} \dots, \gamma_{c(a2)}, ..$$

$\gamma_{1(ab)}\gamma_{2(ab)},..\gamma_{c(ab)}]'$

$$G=\begin{bmatrix}0 & 1'_a & 0_{1\times ab} & 0_{1\times abc}\\ 0_{a\times 1} & 0_{a\times a} & I_a\otimes 1'_b & 0_{a\times abc}\\ 0_{ab\times 1} & 0_{ab\times a} & 0_{ab\times ab} & I_{ab}\otimes 1'_c\end{bmatrix}$$

1_n is nx1 unit matrix, I_n is nxn identity matrix, and $\otimes$ is KroneckerProduct[2, 4, 13].

Model (3) is not full column rank and has constrained on it parameters. To transform the constrained model (3) into unconstrained model, method of model reduction [5] is used. First it is to transform the parameter θ into θ_1 by using permutation matrix T as follows:

$$\theta_1 = T\theta \tag{4}$$

and $T'T = I_{a+ab+abc+1}$

From (4) we have

$$T=\begin{bmatrix}0 & (1\quad 0_{1\times(a-1)}) & 0_{1\times ab} & 0_{1\times abc}\\ 0_{a\times 1} & 0_{a\times a} & I_a\otimes(1\quad 0_{1\times(b-1)}) & 0_{a\times abc}\\ 0_{ab\times 1} & 0_{ab\times a} & 0_{ab\times ab} & I_{ab}\otimes(1\quad 0_{1\times(c-1)})\\ 0_{(a-1)\times 1} & (0_{(a-1)\times 1} I_{(a-1)}) & 0_{(a-1)\times ab} & 0_{(a-1)\times abc}\\ 0_{a(b-1)\times 1} & 0_{a(b-1)\times a} & I_a\otimes(0_{(b-1)\times 1} I_{(b-1)}) & 0_{a(b-1)\times abc}\\ 0_{ab(c-1)\times 1} & 0_{ab(c-1)\times a} & 0_{ab(c-1)\times ab} & I_{ab}\otimes(0_{(c-1)\times 1} I_{(c-1)})\\ 1 & 0_{1\times a} & 0_{1\times ab} & 0_{1\times abc}\end{bmatrix}$$

$$\theta_1 = [\tau_1,\beta_{1(1)},..\beta_{1(a)},\gamma_{1(11)},..\gamma_{1(1b)},..\gamma_{1(a1)},..,\gamma_{1(ab)},\tau_{\{1\}},\beta_{\{1(1)\}},..,\beta_{\{1(a)\}},\gamma_{\{1(11)\}}\dots,\gamma_{\{1(1b)\}},..,\gamma_{\{1(a1)\}},..,\gamma_{\{1(ab)\}},\mu]'$$

where

$\tau_{\{1\}} = (\tau_2,\tau_3 \dots\dots,\tau_a)'$

$\beta_{\{1(1)\}} = (\beta_{2(1)},\beta_{3(1)} \dots\dots,\beta_{b(1)})'$

$\beta_{\{1(a)\}} = (\beta_{2(a)},\beta_{3(a)} \dots\dots,\beta_{b(a)})'$

$\gamma_{\{1(11)\}} = (\gamma_{2(11)},\gamma_{3(11)} \dots\dots,\gamma_{c(11)})'$

$\gamma_{\{1(1b)\}} = (\gamma_{2(1b)},\gamma_{3(1b)} \dots\dots,\gamma_{c(1b)})'$

$\gamma_{\{1(a1)\}} = (\gamma_{2(a1)},\gamma_{3(a1)} \dots\dots,\gamma_{c(a1)})'$

$\gamma_{\{1(ab)\}} = (\gamma_{2(ab)},\gamma_{3(ab)} \dots\dots,\gamma_{c(ab)})'$

Now (3) become

$$Y = X_1\theta_1 + \varepsilon \tag{5}$$

Constrained $G_1\theta_1 = 0$

where $X_1 = XT'$, $G_1 = GT'$ and $\theta_1 = T\theta$.

So we have

$$G_1=\begin{bmatrix}1 & 0'_a & 0'_{ab} & 1'_{(a-1)} & 0'_{a(b-1)} & 0'_{ab(c-1)} & 0\\ 0_a & I_a & 0_{a\times ab} & 0_{a\times(a-1)} & I_a\otimes 1'_{(b-1)} & 0_{a\times ab(c-1)} & 0_a\\ 0_{ab} & 0_{ab\times a} & I_{ab} & 0_{ab\times(a-1)} & 0_{ab\times a(b-1)} & I_{ab}\otimes 1'_{(c-1)} & 0_{ab}\end{bmatrix}$$

$$X_1 = [\Psi_1 \quad \Psi_2 \quad \Psi_3\,\Psi_4 \quad \Psi_5 \quad \Psi_6 \;\; 1_{abcn}]$$

where

$\Psi_1 = \begin{pmatrix}1_{bcn}\\ 0_{(a-1)bcn}\end{pmatrix}$

$\Psi_2 = I_a\otimes\begin{pmatrix}1_{cn}\\ 0_{(b-1)cn}\end{pmatrix}$

$\Psi_3 = I_{ab}\otimes\begin{pmatrix}1_n\\ 0_{(c-1)n}\end{pmatrix}$

$\Psi_4 = \begin{pmatrix}0_{bcn\times(a-1)}\\ I_{(a-1)}\otimes 1_{bcn}\end{pmatrix}$

$\Psi_5 = I_a\otimes\begin{pmatrix}0_{cn\times(b-1)}\\ I_{(b-1)}\otimes 1_{cn}\end{pmatrix}$

$\Psi_6 = I_{ab}\otimes\begin{pmatrix}0_{n\times(c-1)}\\ I_{(c-1)}\otimes 1_n\end{pmatrix}$

Then we partition X_1, G_1, and θ_1 as follow

$X_1 = [X_{11} \quad X_{12}]$

$G_1 = [G_{11} \quad G_{12}]$

$\theta_1 = [\theta_{11} \quad \theta_{12}]'$

where

$$X_{11} = [\Psi_1 \quad \Psi_2 \quad \Psi_3] \tag{6}$$

$$X_{12} = [\Psi_4 \quad \Psi_5 \quad \Psi_6 1_{abcn}] \tag{7}$$

$$G_{11}=\begin{bmatrix}1 & 0'_a & 0'_{ab}\\ 0_a & I_a & 0_{a\times ab}\\ 0_{ab} & 0_{ab\times a} & I_{ab}\end{bmatrix} \tag{8}$$

$$G_{12}=\begin{bmatrix}1'_{(a-1)} & 0'_{a(b-1)} & 0'_{ab(c-1)} & 0\\ 0_{a\times(a-1)} & I_a\otimes 1'_{(b-1)} & 0_{a\times ab(c-1)} & 0_a\\ 0_{ab\times(a-1)} & 0_{ab\times a(b-1)} & I_{ab}\otimes 1'_{(c-1)} & 0_{ab}\end{bmatrix} \tag{9}$$

$$\theta_{11} = [\tau_1,\beta_{1(1)},\dots\beta_{1(a)},\gamma_{1(11)},\dots\gamma_{1(1b)},\dots,\gamma_{1(a1)},\dots,\gamma_{1(ab)}]'$$

$$\theta_{12} = [\tau_{\{1\}},\beta_{\{1(1)\}},..\beta_{\{1(a)\}},\gamma_{\{1(11)\}},..\gamma_{\{1(1b)\}},..\gamma_{\{1(a1)\}},..\gamma_{\{1(ab)\}},\mu]' \tag{10}$$

From (5) to (10), we apply the method of model reduction [5], then we have

$$Y = X_{1r}\theta_{1r} + \varepsilon \tag{11}$$

where

$X_{1r} = [X_{12} - X_{11}G_{11}^{-1}G_{12}]$
$\theta_{1r} = \theta_{12}$

To find the matrix X_{1r} first it need to find the matrix G_{11}^{-1}. From (8), G_{11} is an identity matrix with the order $+a+1$, so that

$X_{11}G_{11}^{-1}G_{12}$=$X_{11}G_{12}$ and

$$X_{11}G_{12} = \left[\Psi_1(1'_{(a-1)}) \quad \Psi_2(I_a \otimes 1'_{(b-1)}) \quad \Psi_3(I_{ab} \otimes 1'_{(c-1)}) \quad 0_{abcn}\right] \tag{12}$$

From (7), (11) and (12) it is found that

$$X_{1r} = \left[\begin{pmatrix} -(1'_{(a-1)} \otimes 1_{bcn}) \\ I_{(a-1)} \otimes 1_{bcn} \end{pmatrix} \quad I_a \otimes \begin{pmatrix} -(1'_{(b-1)} \otimes 1_{cn}) \\ I_{(b-1)} \otimes 1_{cn} \end{pmatrix} \quad I_{ab} \otimes \begin{pmatrix} -(1'_{(c-1)} \otimes 1_n) \\ I_{(c-1)} \otimes 1_n \end{pmatrix} \quad 1_{abcn}\right] \tag{13}$$

The parameter vector $\theta_{1r} = \theta_{12}$ is given by (10).

Lemma 1
Model (11) is full column rank.

Proof

To prove that the model (11) is full column rank, it is sufficient to show that the rank of matrix X_{1r} is equal to abc
$rank\ (X_{1r}) = rank\ (X'_{1r}X_{1r})$
$X'_{1r}X_{1r} = Block\ diag(\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4)$

where:

$\Gamma_1 = [J_{(a-1)} + I_{(a-1)}]bcn$
$\Gamma_2 = I_a \otimes [J_{(b-1)} + I_{(b-1)}]cn$
$\Gamma_3 = I_{ab} \otimes [J_{(c-1)} + I_{(c-1)}]n$
$\Gamma_4 = 1'_{abcn}1_{abcn} = abcn$

Since $\Gamma_1, \Gamma_2, \Gamma_3,$ and Γ_4 are nonsingular matrices, then $X'_{1r}X_{1r}$ is nonsingular and has the rank *abc.*

3. ESTIMATION OF PARAMETER

Model (11) is unconstrained and has full column rank. By using general Gauss Markov theorem [3, 13], the estimation of the parameters (11) is unbiased and has optimal property.
The estimation of θ_{1r} is

$$\hat{\theta}_{1r} = (X'_{1r}X_{1r})^{-1}X'_{1r}Y$$
$$= X_{1r}^{-}Y \tag{14}$$

where $X_{1r}^{-} = (X'_{1r}X_{1r})^{-1}X'_{1r}$
$E(\hat{\theta}_{1r}) = E(X_{1r}^{-}Y)$
$= X_{1r}^{-}E(Y)$
$= X_{1r}^{-}X_{1r}\theta_{1r}$
$$= \theta_{1r} \tag{15}$$

The variance of $\hat{\theta}_{1r}$ is

$$\begin{aligned} Var\,(\hat{\theta}_{1r}) &= E\{[\hat{\theta}_{1r} - E(\hat{\theta}_{1r})][\hat{\theta}_{1r} - E(\hat{\theta}_{1r})]'\} \\ &= E\{[X_{1r}^{-}Y - \theta_{1r}][X_{1r}^{-}Y - \theta_{1r}]'\} \\ &= E\{[X_{1r}^{-}(X_{1r}\theta_{1r} + \varepsilon) - \theta_{1r}]\,[X_{1r}^{-}(X_{1r}\theta_{1r} + \varepsilon) - \theta_{1r}]'\} \\ &= E\{[X_{1r}^{-}\varepsilon]\,[X_{1r}^{-}\varepsilon]'\} \\ &= X_{1r}^{-}E(\varepsilon\varepsilon')X'^{-}_{1r} \\ &= \sigma^2(X_{1r}'X_{1r})^{-1} \end{aligned} \tag{16}$$

Therefore $\hat{\theta}_{1r}$ is normally distributed with mean θ_{1r} and variance

$$\sigma^2(X_{1r}'X_{1r})^{-1} \tag{17}$$

The unbiased estimator of σ^2 is

$$\hat{\sigma}^2 = \frac{1}{abc(n-1)}Y'[I - X_{1r}X_{1r}^{-}]Y. \tag{18}$$

In the next step below, we are going to check the characteristic of the estimators. We will check whether the estimator satisfied the criteria of Uniformly Minimum Variance Unbiased Estimator (UMVUE) [3]. Namely, we are going to check:

(i) $E(\hat{\theta}_{1r}) = \theta_{1r}$, namely $\hat{\theta}_{1r}$ is unbiased estimator of θ_{1r}.
(ii) $Var(\hat{\theta}_{1r}) \leq Var(\theta^*_{1r})$, where θ^*_{1r} is the other estimator of θ_{1r}

Proof

To prove (i) it is sufficient to show that $E(\hat{\theta}_{1r}) = \theta_{1r}$ and the equation (15) has proved it, that is $\hat{\theta}_{1r}$ is unbiased estimator of θ_{1r}.

To prove (ii), let θ^*_{1r} is other unbiased estimator of θ_{1r}, then we will show that
$Var(\hat{\theta}_{1r}) \leq Var(\theta^*_{1r})$.
Let θ^*_{1r} is written in the form
$\theta^*_{1r} = (X_{1r}^{-} + A)\,Y$
where A is abc x abcn matrix, so that

$$\begin{aligned} E[\theta^*_{1r}] &= E[(X_{1r}^{-} + A)\,Y] \\ &= E[(X_{1r}^{-} + A)\,Y] \\ &= (X_{1r}^{-} + A)E[\,Y] \\ &= (X_{1r}^{-} + A)X_{1r}\theta_{1r} \\ &= (X_{1r}^{-}X_{1r} + AX_{1r})\theta_{1r} \end{aligned}$$

$$= (I + AX_{1r})\theta_{1r}$$

ince θ^*_{1r} is unbiased estimator, then
$E(\theta^*_{1r}) \quad = \theta_{1r}$
$(I + AX_{1r})\theta_{1r} = \theta_{1r}$.
Therefore $(I + AX_{1r}) = I$ and $AX_{1r} = 0$

$$\begin{aligned}
Var(\theta^*_{1r}) &= E\{[\theta^*_{1r} - E(\theta^*_{1r})][\theta^*_{1r} - E(\theta^*_{1r})]'\} \\
&= E\{[(X^-_{1r} + A)\,Y - (I + AX_{1r})\theta_{1r}] \\
&\quad [(X^-_{1r} + A)\,Y - (I + AX_{1r})\theta_{1r}]'\} \\
&= E\{[(X^-_{1r} + A)\,(X_{1r}\theta_{1r} + \varepsilon) - \theta_{1r}] \\
&\quad [(X^-_{1r} + A)\,(X_{1r}\theta_{1r} + \varepsilon) - \theta_{1r}]'\} \\
&= E\{[(X^-_{1r}X_{1r}\theta_{1r} + X^-_{1r}\varepsilon + AX_{1r}\theta_{1r} + A\varepsilon) - \theta_{1r}\,] \\
&\quad [(X^-_{1r}X_{1r}\theta_{1r} + X^-_{1r}\varepsilon + AX_{1r}\theta_{1r} + A\varepsilon) - \theta_{1r}]'\} \\
&= E\{[\theta_{1r} + (X^-_{1r} + A)\varepsilon - \theta_{1r}\,][\theta_{1r} + (X^-_{1r} + A)\varepsilon - \theta_{1r}]'\} \\
&= E\{[(X^-_{1r} + A)\varepsilon\,][(X^-_{1r} + A)\varepsilon]'\} \\
&= E\{(X^-_{1r} + A)(\varepsilon\,\varepsilon')(X^-_{1r} + A)'\} \\
&= \sigma^2[X^-_{1r}X^{-'}_{1r} + X^-_{1r}A + AX^{-'}_{1r} + AA'] \\
&= \sigma^2[(X_{1r}'X_{1r})^{-1} + AA'] \\
&= \sigma^2(X_{1r}'X_{1r})^{-1} + \sigma^2AA' \\
&= Var\left(\hat{\theta}_{1r}\right) + \sigma^2AA' \qquad (19)
\end{aligned}$$

Therefore (ii) has been proven, $Var(\hat{\theta}_{1r}) \le Var(\theta^*_{1r})$.

4. TESTING OF HYPOTHESES

From model (11), we can test some function of parameters by using Generalized Likelihood Ratio Test [3], some hypotheses and the statistics to test the hypotheses are presented in the theorems below.

Theorem 1

In the unconstrained linear model (11), Λ_1 is a statistics test of generalized likelihood ratio (GLR) to test the hypothesis

$H_0\colon H_1\theta_{1r} = h_1$ against $H_a\colon H_1\theta_{1r} \ne h_1$ (20)

Where

$$H_1 = [(1_{(a-2)} \quad -I_{(a-2)}) \quad 0_{(a-2)\times a(b-1)} \quad 0_{(a-2)\times ab(c-1)} \quad 0_{(a-2)\times 1}]$$

$$h_1 = 0_{[a(bc-b-1)-1]\times 1}$$

and the statistic test is

$$\Lambda_1 = \frac{(H_1\hat{\theta}_{1r})'\left[H_1(X'_{1r}X_{1r})^{-1}H_1'\right]^{-1}(H_1\hat{\theta}_{1r})/df1}{Y'[I - X_{1r}X^-_{1r}]Y/df2} \qquad (21)$$

where $df1$ is the rank of matrix H_1, $rank(H_1) = a - 2$ and $df2$ is the rank of matrix $[I - X_{1r}X^-_{1r}]$, $rank([I - X_{1r}X^-_{1r}]) = abc(n-1)$. *Under* H_0 Λ_1 has a distribution $F_{(a-2,abc(n-1))}$, and the criteria test is

Reject H_0 if $\Lambda_1 \ge F_{\left(\alpha:(a-2),(abc(n-1))\right)}$

where $F_{\left(\alpha:(a-2),(abc(n-1))\right)}$ is the upper probability point of the central F-distribution with $a - 2$ and $abc(n-1)$ degrees of freedom.

Proof

From (11) the random error ε is $abcn \times 1$ vector and has $N(0, \Sigma)$, distribution so that Y is random vector $abcn \times 1$ and has $N(X_{1r}\theta_{1r}, \Sigma)$ distribution. To find the distribution of $Y'[I - X_{1r}X^-_{1r}]Y$ the Theorem 2.3.3 ([14], p. 62) is used and we have to show that the matrix $[I - X_{1r}X^-_{1r}]\Sigma$ is idempotent.

Let U_1 is $Y'KY$ where $K = [I - X_{1r}X^-_{1r}]$, since Y has a multivariate distribution, then there exists a matrix C nonsingular such that $C'C = \Sigma$. Define the random variable Z,

$$Z = (C')^{-1}(Y - X_{1r}\theta_{1r}) \qquad (22)$$

then Z has $N(0, I)$ distribution. From (22) we have

$$Y = C'Z + X_{1r}\theta_{1r} \qquad (23)$$

So that

$$\begin{aligned}
Y'KY &= (C'Z + X_{1r}\theta_{1r})'K(C'Z + X_{1r}\theta_{1r}) \\
&= (Z'C + \theta'_{1r}X'_{1r})K(C'Z + X_{1r}\theta_{1r}) \\
&= (Z'C + \theta'_{1r}X'_{1r})C^{-1}CKC'(C')^{-1}(C'Z + X_{1r}\theta_{1r}) \\
&= (Z' + \theta'_{1r}X'_{1r}C^{-1})CKC'(Z + (C')^{-1}X_{1r}\theta_{1r}) \\
&= (Z + (C')^{-1}X_{1r}\theta_{1r})'CKC'(Z + (C')^{-1}X_{1r}\theta_{1r}) \\
&= V'B_1V \qquad (24)
\end{aligned}$$

where $V = (Z + (C')^{-1}X_{1r}\theta_{1r})$ and V has distribution $N((C')^{-1}X_{1r}\theta_{1r}, I)$ also $B_1 = CKC'$

$$\begin{aligned}
B_1 &= B_1B_1 \\
CKC' &= CKC'CKC' \\
CKC' &= CK\Sigma KC' \\
C^{-1}CKC'C &= C^{-1}CK\Sigma KC'C \\
K\Sigma &= K\Sigma K\Sigma \qquad (25)
\end{aligned}$$

The equation (25) shows that $[I - X_{1r}X^-_{1r}]\Sigma$ is idempotent. Since Σ is nonsingular, then

$$rank(K\Sigma) = rank[K] = rank[I - X_{1r}X^-_{1r}]\,.$$

Since $[I - X_{1r}X^-_{1r}]$ is idempotent, then

$$\begin{aligned}
rank(K\Sigma) &= tr[I - X_{1r}X^-_{1r}] \\
&= tr(I) - tr(X_{1r}X_{1r}{}^-) \\
&= tr(I) - tr(X_{1r}{}^-X_{1r}) \\
&= abc(n-1)
\end{aligned}$$

Based on Theorem 4.4.3 ([3], p.135), U_1 has a Chi-Square distribution with degrees of freedom $abc(n-1)$ with noncentrality parameter equals to zero,

$$(\lambda = 0). \qquad (26)$$

Let $U_2 = (H_1\hat{\theta}_{1r})'\left[H_1(X'_{1r}X_{1r})^{-1}H_1'\right]^{-1}(H_1\hat{\theta}_{1r})$ (27)

Define $\varphi_1 = H_1\theta_{1r}$, then the estimator $\varphi_1 = \hat{\varphi}_1$ is

$$\hat{\varphi}_1 = H_1\hat{\theta}_{1r} \qquad (28)$$

Substitute (28) into (27), we have

$$\hat{\varphi}'_1\left[H_1(X'_{1r}X_{1r})^{-1}H_1'\right]^{-1}\hat{\varphi}_1 \qquad (29)$$

$\hat{\varphi}_1$ is a $(a-2) \times 1$ random vector and has distribution

$$N\left(\varphi_1, \sigma^2H_1(X'_{1r}X_{1r})^{-1}H_1'\right) \qquad (30)$$

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From (30) by using Corollary 4.2.1.4 ([3], p. 127) then

$$\frac{1}{\sigma^2}\hat{\varphi}_1'\left[H_1(X_{1r}'X_{1r})^{-1}H_1'\right]^{-1}\hat{\varphi}_1 = \frac{U_2}{\sigma^2} \quad (31)$$

has Chi-Square distribution, with the degrees of freedom (a-2).so that U_2 has Chi-Square distribution with $(a-2)$ degrees of freedom.

Next we will show that U_1 and U_2 are independent.
Let $L_1 = \left[H_1(X_{1r}'X_{1r})^{-1}H_1'\right]^{-1}$ and to show that U_1 and U_2 independent, we have to show that $K\Sigma L_1 = 0$.

$$\begin{aligned} K\Sigma L_1 &= [I - X_{1r}X_{1r}^{-}]\sigma^2 I[(H_1X_{1r}^{-})(H_1X_{1r}^{-})']^{-1} \\ &= \sigma^2 I[I - X_{1r}X_{1r}^{-}][(H_1X_{1r}^{-})'^{-}(H_1X_{1r}^{-})^{-}] \\ &= \sigma^2 I[(H_1X_{1r}^{-})'^{-}(H_1X_{1r}^{-})^{-} - X_{1r}X_{1r}^{-}(H_1X_{1r}^{-})'^{-}(H_1X_{1r}^{-})^{-}] \\ &= \sigma^2 I[(H_1X_{1r}^{-})'^{-}(H_1X_{1r}^{-})^{-} - X_{1r}X_{1r}^{-}X_{1r}'H_1'^{-}(H_1X_{1r}^{-})^{-}] \\ &= \sigma^2 I[(H_1X_{1r}^{-})'^{-}(H_1X_{1r}^{-})^{-} - X_{1r}H_1'(H_1X_{1r}^{-})^{-}] \\ &= \sigma^2 I[(H_1X_{1r}^{-})'^{-}(H_1X_{1r}^{-})^{-} - (H_1X_{1r}^{-})'^{-}(H_1X_{1r}^{-})^{-}] \\ &= \sigma^2 I[0] = 0 \end{aligned} \quad (32)$$

Since $K\Sigma L_1 = 0$, we conclude that U_1 and U_2 are independent.

From (26), (31) and (32), Λ_1 is the ratio of two independent Chi-square distributions, therefore under H_0 Λ_1 has $F_{(a-2,abc(n-1))}$ distribution.

Theorem 2

From general linear model (11), Λ_2 is a *generalized likelihood ratio* (GLR) test for testing the hypothesis

$$H_0: H_2\theta_{1r} = h_2 \text{ against } H_a: H_2\theta_{1r} \neq h_2 \quad (33)$$

where

$$H_2 = [0_{a(b-2)\times(a-1)} \quad I_a \otimes (1_{(b-2)} \quad -I_{(b-2)}) \quad 0_{a(b-2)\times ab(c-1)} \quad 0_{a(b-2)\times 1}]$$

$$h_2 = 0_{[a(bc-b-1)-1]\times 1}$$

and the statistic test is

$$\Lambda_2 = \frac{(H_2\hat{\theta}_{1r})'\left[H_2(X_{1r}'X_{1r})^{-1}H_2'\right]^{-1}(H_2\hat{\theta}_{1r})/df3}{Y'[I - X_{1r}X_{1r}^{-}]Y/df2} \quad (34)$$

where $df3$ is the rank of the matrix H_2, $rank[H_2] = a(b-2)$. *Under* H_0 Λ_2 has an $F_{(a(b-2),abc(n-1))}$ distribution. The criteria test is
Reject H_0 if $\Lambda_2 \geq F_{(\alpha:a(b-2),(abc(n-1)))}$
where $F_{(\alpha:a(b-2),(abc(n-1)))}$ is the upper probability point of the central F-distribution with $a(b-2)$ and $abc(n-1)$degrees of freedom.

Proof

$$\text{Let } U_3 = (H_2\hat{\theta}_{1r})'\left[H_2(X_{1r}'X_{1r})^{-1}H_2'\right]^{-1}(H_2\hat{\theta}_{1r}) \quad (35)$$

Define $\varphi_2 = H_2\theta_{1r}$, then the estimator of $\varphi_2 = \hat{\varphi}_2$ is

$$\hat{\varphi}_2 = H_2\hat{\theta}_{1r} \quad (36)$$

Then (35) becomes

$$\hat{\varphi}_2'\left[H_2(X_{1r}'X_{1r})^{-1}H_2'\right]^{-1}\hat{\varphi}_2 \quad (37)$$

$\hat{\varphi}_2$ is $a(b-2)\times 1$ random vector and has distribution

$$N\left(\varphi_2, \sigma^2 H_2(X_{1r}'X_{1r})^{-1}H_2'\right) \quad (38)$$

From (38) by using Corollary 4.2.1.4 ([3], p. 127) then

$$\frac{1}{\sigma^2}\hat{\varphi}_2'\left[H_2(X_{1r}'X_{1r})^{-1}H_2'\right]^{-1}\hat{\varphi}_2 = \frac{U_3}{\sigma^2} \quad (39)$$

has a chi-squares distribution with degrees of freedom $a(b-2)$, so U_3 also has chi-squares distribution.

Next we will show that U_1 and U_3 are independent.
Let $L_2 = \left[H_2(X_{1r}'X_{1r})^{-1}H_2'\right]^{-1}$

$$\begin{aligned} K\Sigma L_2 &= [I - X_{1r}X_{1r}^{-}]\sigma^2 I[(H_2X_{1r}^{-})(H_2X_{1r}^{-})']^{-1} \\ &= \sigma^2 I[I - X_{1r}X_{1r}^{-}][(H_2X_{1r}^{-})'^{-}(H_2X_{1r}^{-})^{-}] \\ &= \sigma^2 I[(H_2X_{1r}^{-})'^{-}(H_2X_{1r}^{-})^{-} - X_{1r}X_{1r}^{-}(H_2X_{1r}^{-})'^{-}(H_2X_{1r}^{-})^{-}] \\ &= \sigma^2 I[(H_2X_{1r}^{-})'^{-}(H_2X_{1r}^{-})^{-} - X_{1r}X_{1r}^{-}X_{1r}'H_2'^{-}(H_2X_{1r}^{-})^{-}] \\ &= \sigma^2 I[(H_2X_{1r}^{-})'^{-}(H_2X_{1r}^{-})^{-} - X_{1r}H_2'(H_2X_{1r}^{-})^{-}] \\ &= \sigma^2 I[(H_2X_{1r}^{-})'^{-}(H_2X_{1r}^{-})^{-} - (H_2X_{1r}^{-})'^{-}(H_2X_{1r}^{-})^{-}] \\ &= \sigma^2 I[0] = 0 \end{aligned} \quad (40)$$

Since $K\Sigma L_2 = 0$, then U_1 and U_3 are independent.

From (26), (39) and (40) Λ_2 is the ratio of two independent Chi-Square distributions. Therefore under H_0 Λ_2 has $F_{(a-2,abc(n-1))}$ distribution.

Theorem 3

General linear model (11), Λ_3 is Generalized Likelihood Ratio (GLR) test for testing the hypothesis

$$H_0: H_3\theta_{1r} = h_3 \text{ against } H_a: H_3\theta_{1r} \neq h_3 \quad (41)$$

where

$$H_3 = [0_{ab(c-2)\times(a-1)} \quad 0_{(a-2)\times a(b-1)} \quad I_{ab} \otimes (1_{(c-2)} \quad -I_{(c-2)}) \quad 0_{ab(c-2)}]$$

$$h_3 = 0_{ab(c-2)\times 1}$$

The statistic test is

$$\Lambda_3 = \frac{(H_3\hat{\theta}_{1r})'\left[H_3(X_{1r}'X_{1r})^{-1}H_3'\right]^{-1}(H_3\hat{\theta}_{1r})/df4}{Y'[I - X_{1r}X_{1r}^{-}]Y/df2} \quad (42)$$

where $df4$ is the rank of matrix H_3, $rank(H_3) = ab(c-2)$. *Under* H_0, Λ_3 has $F_{(ab(c-2),abc(n-1))}$ distribution and the criteria test is
Reject H_0 if $\Lambda_3 \geq F_{(\alpha:ab(c-2),(abc(n-1)))}$
where $F_{(\alpha:ab(c-2),(abc(n-1)))}$ is the upper probability point of the central F-distribution with $ab(c-2)$ and $abc(n-1)$ degrees of freedom.

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Proof

Let U_4 is $\left(H_3\hat{\theta}_{1r}\right)'\left[H_3(X_{1r}'X_{1r})^{-1}H_3'\right]^{-1}\left(H_3\hat{\theta}_{1r}\right)$ (43)

Define $\varphi_3 = H_3\theta_{1r}$, then the estimator of $\varphi_3 = \hat{\varphi}_3$ is

$$\hat{\varphi}_3 = H_3\hat{\theta}_{1r}. \quad (44)$$

Substitute the equation (44) so that the equation (43) becomes

$$\hat{\varphi}_3'\left[H_3(X_{1r}'X_{1r})^{-1}H_3'\right]^{-1}\hat{\varphi}_3 \quad (45)$$

$\hat{\varphi}_3$ is $a(b-2)\times 1$ random vector and has multivariate normal distribution $N\left(\varphi_3, \sigma^2 H_3(X_{1r}'X_{1r})^{-1}H_3'\right)$. (46)

From (46) by using Corollary 4.2.1.4 ([3], p. 127) then

$$\frac{1}{\sigma^2}\hat{\varphi}_3'\left[H_3(X_{1r}'X_{1r})^{-1}H_3'\right]^{-1}\hat{\varphi}_3 = \frac{U_4}{\sigma^2}$$

has Chi-Square distribution, so that U_4 also has Chi Square distribution with $ab(c-2)$ degrees of fredom. (47)

Next we will show that U_1 and U_4 are independent.

Let $L_3 = \left[H_3(X_{1r}'X_{1r})^{-1}H_3'\right]^{-1}$

$$\begin{aligned} K\Sigma L_3 &= [I - X_{1r}X_{1r}^{-}]\sigma^2 I[(H_3X_{1r}^{-})(H_3X_{1r}^{-})']^{-1} \\ &= \sigma^2 I[I - X_{1r}X_{1r}^{-}][(H_3X_{1r}^{-})'^{-}(H_3X_{1r}^{-})^{-}] \\ &= \sigma^2 I[(H_3X_{1r}^{-})'^{-}(H_3X_{1r}^{-})^{-} - X_{1r}X_{1r}^{-}(H_3X_{1r}^{-})'^{-}(H_3X_{1r}^{-})^{-}] \\ &= \sigma^2 I[(H_3X_{1r}^{-})'^{-}(H_3X_{1r}^{-})^{-} - X_{1r}X_{1r}^{-}X_{1r}'H_3'^{-}(H_3X_{1r}^{-})^{-}] \\ &= \sigma^2 I[(H_3X_{1r}^{-})'^{-}(H_3X_{1r}^{-})^{-} - X_{1r}H_3'(H_3X_{1r}^{-})^{-}] \\ &= \sigma^2 I[(H_3X_{1r}^{-})'^{-}(H_3X_{1r}^{-})^{-} - (H_3X_{1r}^{-})'^{-}(H_3X_{1r}^{-})^{-}] \\ &= \sigma^2 I[0] = 0. \end{aligned} \quad (48)$$

Since $K\Sigma L_3 = 0$, U_1 and U_4 are independent. From (26), (47) and (48), Λ_3 is the ratio of two independent Chi-Square distributions, so under H_0, Λ_3 has $F_{(ab(c-2),abc(n-1))}$ distribution.

5. RATIO OF LINEAR FUNCTION OF PARAMETERS

To build the confidence interval of the ratio of linear function of parameters θ_{1r} in model (11) is as follows:

$$\rho = \frac{M'\theta_{1r}}{N'\theta_{1r}} \quad (49)$$

where M and N are abcx 1 known vector .

Note that:

$$\Omega = \frac{M'\hat{\theta}_{1r} - \rho N'\hat{\theta}_{1r}}{\left[\hat{\sigma}\{M'(X_{1r}'X_{1r})^{-1}M - 2\rho M'(X_{1r}'X_{1r})^{-1}N + \rho^2 N'(X_{1r}'X_{1r})^{-1}N\}^{1/2}\right]} \quad (50)$$

has t- distribution with $abc(n$-1) degrees of freedom.

$(1-\alpha)100\%$ Confidence interval for ρ can be found by using Fieller's argument [9]. Let P is the probability, then $1-\alpha = P[-\omega \le \Omega \le \omega] = P[Q\rho^2 + R\rho + S \le 0]$

where

$$Q = \left(N'\hat{\theta}_{1r}\right)^2 - \omega^2 N'(X_{1r}'X_{1r})^{-1}N\hat{\sigma}^2 \quad (51)$$

$$R = 2\left[\omega^2 M'(X_{1r}'X_{1r})^{-1}N\hat{\sigma}^2 - \left(M'\hat{\theta}_{1r}\right)\left(N'\hat{\theta}_{1r}\right)\right] \quad (52)$$

$$S = \left(M'\hat{\theta}_{1r}\right)^2 - \omega^2 M'(X_{1r}'X_{1r})^{-1}M\hat{\sigma}^2 \quad (53)$$

Let q, r and s denote the value of observation of the above random variables, then we believe that $(1-\alpha)100\%$ is our confidence that ρcontained by the interval

$$\left[\frac{-r-(r^2-4qs)^{1/2}}{2q}, \frac{-r+(r^2-4qs)^{1/2}}{2q}\right] \quad (54)$$

6. SIMULATION

To conduct the simulation, the software R version 3.1.3 was used. In this simulation, for the model three level nested design model (11), each level of the design is a=3, b=4 and c=3, while the replication we take for n=2, n=10 and n= 30, so the vector parameter is

$$\theta_{1r} = \{\tau_2, \tau_3, \beta_{2(1)}, \beta_{2(2)}, \beta_{2(3)}, \beta_{3(1)}, \beta_{3(2)}, \beta_{3(3)}, \beta_{4(1)}, \beta_{4(2)}, \beta_{4(3)}, \gamma_{2(11)}, \gamma_{2(12)}, \gamma_{2(13)}, \gamma_{2(14)}, \gamma_{2(21)}, \gamma_{2(22)}, \gamma_{2(23)}, \gamma_{2(24)}, \gamma_{2(31)}, \gamma_{2(32)}, \gamma_{2(33)}, \gamma_{2(34)}, \gamma_{3(11)}, \gamma_{3(12)}, \gamma_{3(13)}, \gamma_{3(14)}, \gamma_{3(21)}, \gamma_{3(22)}, \gamma_{3(23)}, \gamma_{3(24)}, \gamma_{3(31)}, \gamma_{3(32)}, \gamma_{3(33)}, \gamma_{3(34)}, \mu\}.$$

In this simulation the samples replication is 1,000 and the parameter vector is set:

θ_{1r}={0.2,0.4,0.2,0.4,0.6,0.8,1,1.2,1.4,1.6,1.8,0.2,0.4,0.6,0.8,1,1.2, 1.4,1.6,1.8,2,2.2,2.4,2.6,2.8,3,3.2,3.4,3.6,3.8,4,4.2,4.4,4.6,4.8,5}. The value for variances is set for $\sigma_1^2 = 1$, and $\sigma_2^2 = 9$. To show the unbiased estimate of the parameter, from the 1,000 replication of samples, we calculate the following:

$$E\left(\hat{\theta}_{1r}\right) = \frac{\hat{\theta}_{1r(1)} + \hat{\theta}_{1r(2)} + \hat{\theta}_{1r(3)} + \ldots\ldots + \hat{\theta}_{1r(1.000)}}{1.000}$$

The results of the simulation show that the estimate value $\hat{\theta}_{1r}$ from the unconstraint model found from the application of the method of model reduction [5] are very close to the values of θ_{1r}. The estimate values for the parameter tau (τ_2, τ_3) are very close to the real values for different replication of *n* (Figure 1, 2 and 3). The estimate values for the parameter beta $(\beta_{2(1)}, \beta_{2(2)}, \beta_{2(3)}, \beta_{3(1)}, \beta_{3(2)}, \beta_{3(3)}, \beta_{4(1)}, \beta_{4(2)}, \beta_{4(3)},)$ are very close to the real values beta for different replication of n (Figures 4, 5 and 6). The estimate values for the parameter gamma $(\gamma_{2(11)}, \gamma_{2(12)}, \gamma_{2(13)}, \gamma_{2(14)}, \gamma_{2(21)}, \gamma_{2(22)}, \gamma_{2(23)}, \gamma_{2(24)}, \gamma_{2(31)}, \gamma_{2(32)}, \gamma_{2(33)}, \gamma_{2(34)}, \gamma_{3(11)}, \gamma_{3(12)}, \gamma_{3(13)}, \gamma_{3(14)}, \gamma_{3(21)}, \gamma_{3(22)}, \gamma_{3(23)}, \gamma_{3(24)}, \gamma_{3(31)}, \gamma_{3(32)}, \gamma_{3(33)}, \gamma_{3(34)})$ are very close to the real values gamma for different replication of n (Figures 7, 8 and 9).

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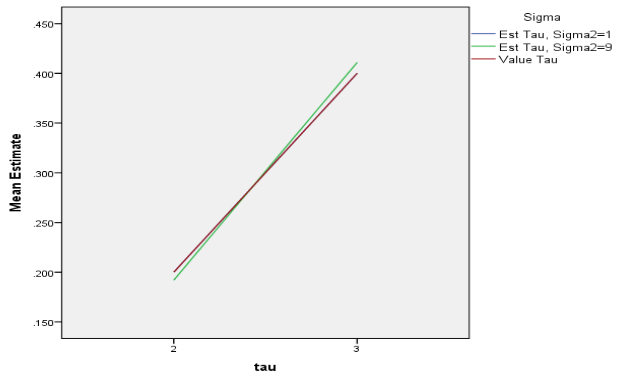

Figure-1. The estimate of Tau (a=3, b=4, c=3, n=2).

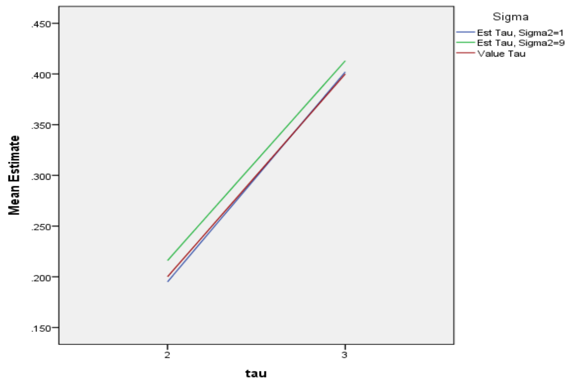

Figure-2. The estimate of Tau (a=3, b=4, c=3, n=10).

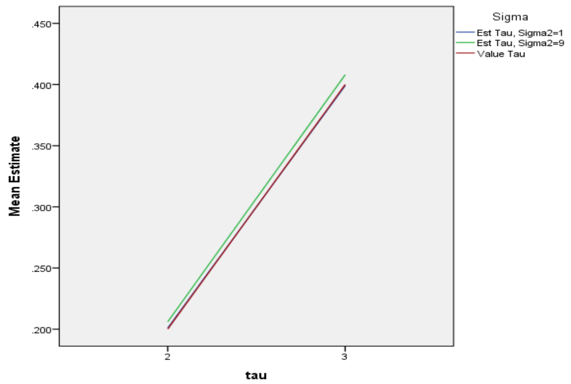

Figure-3. The estimate of Tau (a=3, b=4, c=3, n=30).

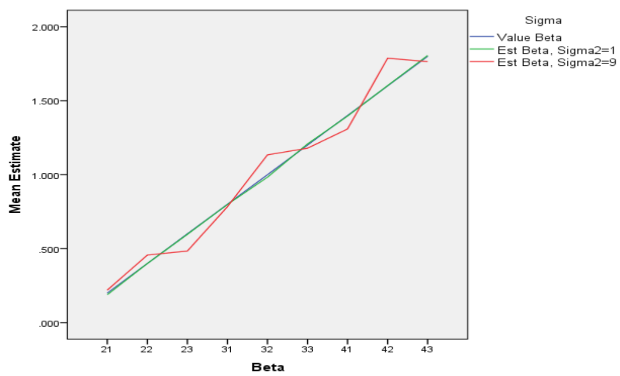

Figure-4. The estimate of Beta (a=3, b=4, c=3, n=2).

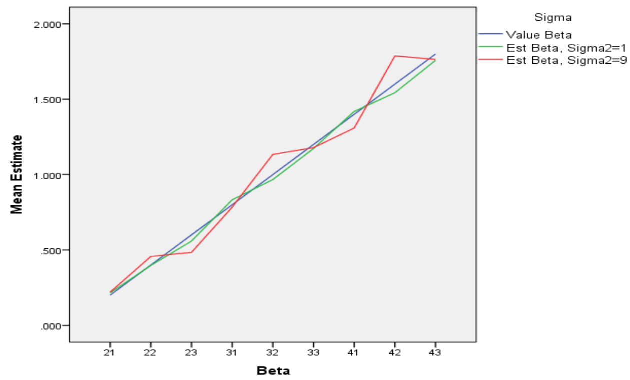

Figure-5. The estimate of Beta (a=3, b=4, c=3, n=10)

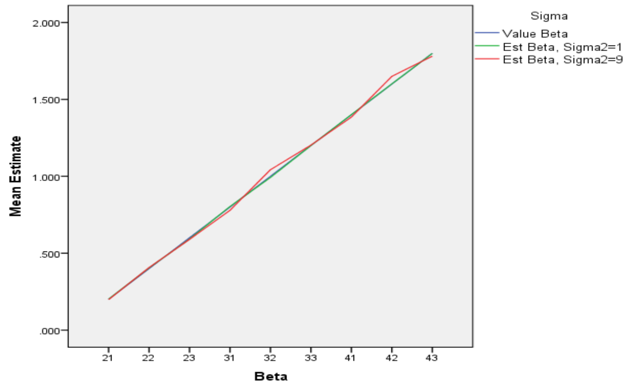

Figure-6. The estimate of Beta (a=3, b=4, c=3, n=30).

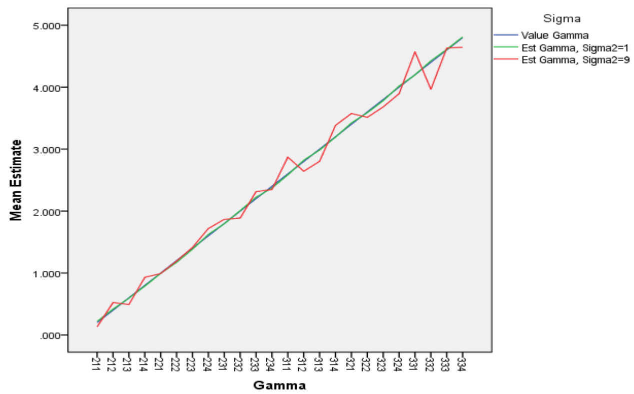

Figure-7. The estimate of Gamma (a=3, b=4, c=3, n=2).

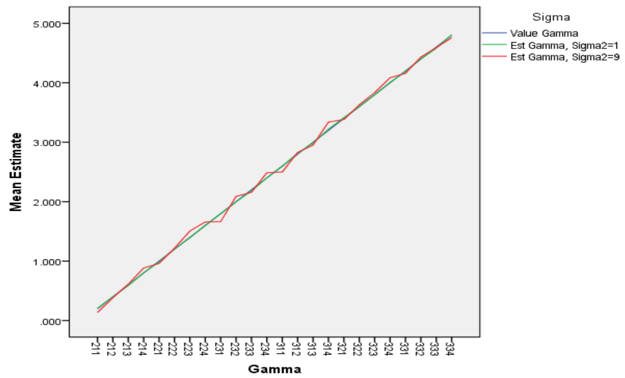

Figure-8. The estimate of Gamma (a=3, b=4, c=3, n=10).

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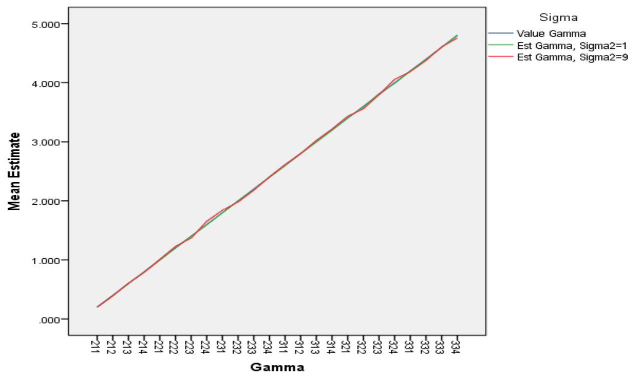

Figure-9. The estimate of Gamma (a=3, b=4, c=3, n=30).

To test the hypotheses related to parameters Tau, Beta and Gamma, we define the null hypotheses as follows:

a) $H_0: \tau_2 = \tau_3$; against $H_a: \tau_2 \neq \tau_3$.

b) $H_0: \beta_{2(1)} = \beta_{2(2)} = \beta_{2(3)} = \beta_{3(1)} = \beta_{3(2)} = \beta_{3(3)} = \beta_{4(1)} = \beta_{4(2)} = \beta_{4(3)}$; against
H_a:at least one $\beta_{j(i)}$ is different from the others.

c) $H_0: \gamma_{2(11)} = \gamma_{2(12)} = \gamma_{2(13)} = \gamma_{2(14)} = \gamma_{2(21)} = \gamma_{2(22)} = \gamma_{2(23)} = \gamma_{2(24)} = \gamma_{2(31)} = \gamma_{2(32)} = \gamma_{2(33)} = \gamma_{2(34)} = \gamma_{3(11)} = \gamma_{3(12)} = \gamma_{3(13)} = \gamma_{3(14)} = \gamma_{3(21)} = \gamma_{3(22)} = \gamma_{3(23)} = \gamma_{3(24)} = \gamma_{3(31)} = \gamma_{3(32)} = \gamma_{3(33)} = \gamma_{3(34)}$
against
H_a:at least one $\gamma_{k(ij)}$ is different from the others.

To evaluate the size of the test in this simulation, 1,000 data set was used and the size of the test and the power of the test are calculated for different setting of the parameters. In this simulation we set some different values of σ^2, namely $\sigma^2 = 2$, $\sigma^2 = 4$, and $\sigma^2 = 6$ and different number of replication *n*=2, 6 and 10.The size of the test is given in the following table.

Table-1. The size of the tests under Ho for hypotheses a, b and c.

Hyphoteses	σ^2	$n = 2$	$n = 6$	$n = 10$
a	2	0.056	0.051	0.054
	4	0.055	0.053	0.049
	6	0.051	0.061	0.044
b	2	0.057	0.055	0.053
	4	0.052	0.049	0.052
	6	0.041	0.048	0.042
c	2	0.056	0.049	0.060
	4	0.060	0.051	0.034
	6	0.041	0.048	0.045

In the simulation we set the size of the tests 0.05, and based on the results of the simulation, the size of the test for different values of $\sigma^2$and *n* are very close to 0.05. From the criteria of Pearson and Please [15], for the size of the test 0.05, the result between 0.03 and 0.07 are within the acceptable range (unbiased). An unbiased test of size α has a power function less than or equal to α for all $\theta_{ir} \in \Theta_{ir(Ho)}$, where $\Theta_{ir(Ho)}$ is a parameter space under Ho, and greater than or equal to α for all $\theta_{ir} \in \Theta_{ir(Ha)}$, where $\Theta_{ir(Ha)}$ is a parameter space under Ha [16, 17].The results in Table-1, Figure-10, Figure-11, and Figure-12 show that they are fulfil the criteria above. So the tests are unbiased.

The graph of the size and power of the test for the three hypotheses are given in the following figures.

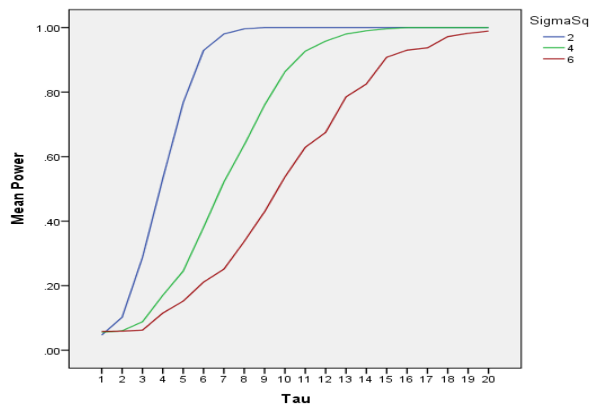

Figure-10. The size and power of the test for hypothesis a.

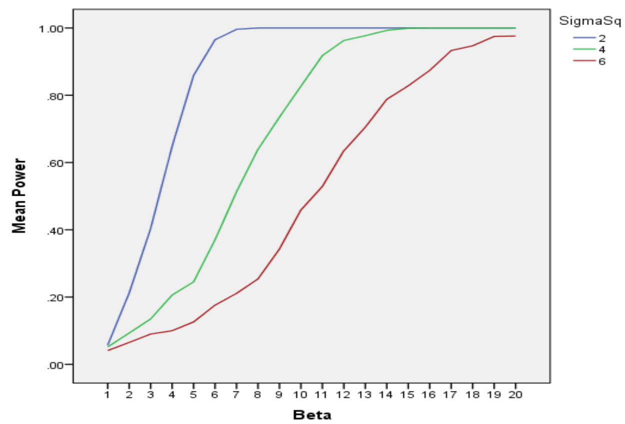

Figure-11. The size and power of the test for hypothesis b.

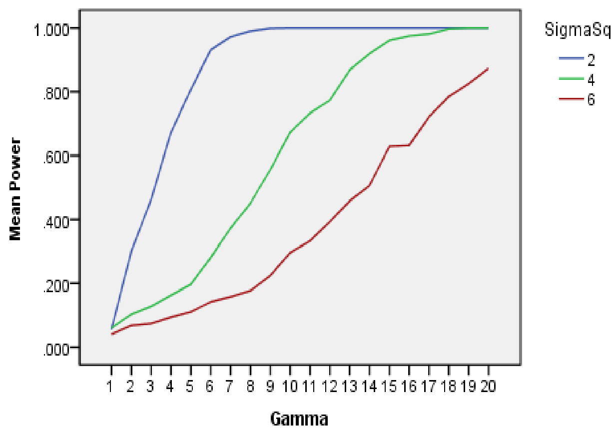

Figure-12. The size and power of the test for hypothesis c.

For the simulation of the ratio of linear function of parameters (49) and its confidence interval (54), we set the value for each parameter in the vector parameterθ_{1r}. In the simulation we set the linear function of parameters tau, beta, and gamma. We set the values of M and N, where M and N are 36x1 vector constant, such that we can find the real value of the ratio ρ, By using Fieller's theorem [9], 1-α confidence interval for ρ can be calculated. The simulation of 95% confidence interval with 1,000 replication of the samples forρ's the resultsare given below.

Table-2. Confidence Interval (CI) of the Ratio of Linear function of parameters ρ.

No.	Ratio of linear function of parameters	ρ	95%Confidence interval (CI)
1	Tau, τ_i	0.68	(0.6680, 1.3491)
		1.00	(0.6539, 1.7969)
		1.31	(0.5512, 4.4064)
		1.56	(0.5332, 5.9989)
		1.58	(0.5241, 6.4865)
2	Beta, $\beta_{j(i)}$	1.26	(0.6984, 1.6306)
		1.32	(0.6071, 2.3523)
		1.62	(0.5581, 4.0075)
		2.17	(0.5165, 8.5525)
		2.30	(0.5109, 8.9643)
3	Gamma, $\gamma_{k(ij)}$	0.96	(0.6888, 1.6429)
		1.09	(0.6122, 2.0874)
		1.17	(0.5851, 2.7322)
		1.33	(0.5396, 4.1915)
		1.73	(0.4834, 8.9401)

The results of the simulation (Table-2) show that in the 95% confidence interval, all values of the ρ's are contained in the interval.

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