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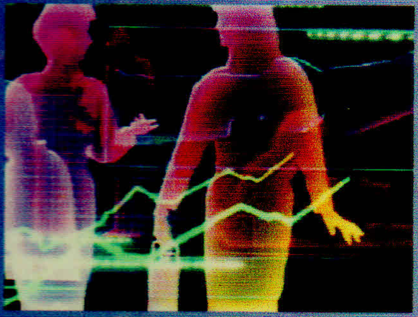
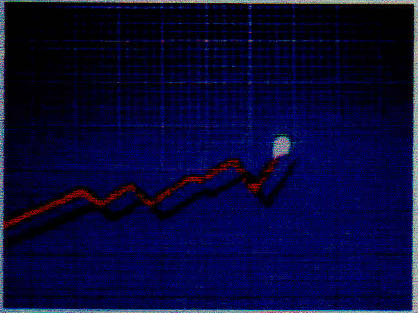
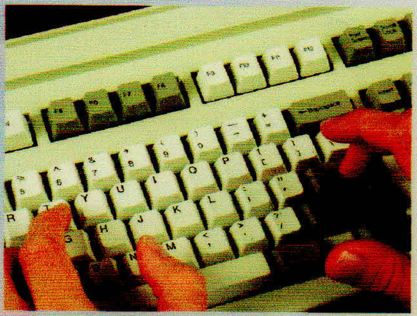
UNIVERSITAS MALAHAYATI INTERNATIONAL CONFERENCE QUANTITATIVE METHODS

USED IN ECONOMICS AND BUSINESS 2008

on October, 15 17th, 2008

FACULTY of ECONOMICS
UNIVERSITAS MALAHAYATI
BANDAR LAMPUNG
INDONESIA

PREFACE

The International Conference on Quantitative Methods Appliend in used economic and Business was conducted by Faculty of Economic, Universitas Malahayati on 15-17 October 2008. The conference was organized by Faculty of Economic Universitas Malahayati and collaborated with Universiti Malaysia Terengganu (UMT) International Islamic University Malaysia (IIUM), and University Putra Malaysia (UPM).

The participants of the conference are about 200 come from more than 20 higher institutions, among others: Universitas Malaysia Perlis, Institut Pertanian Bogor, Universitas of Montenegro, Universitas Bung Hatta, Universitas Putra Malaysia, Universitas of Peshawar Pakistan, Al-Bayt University Al Mafraq- Jordan, Universitas Indonesia, Universitas Gunadarma, Universitas Pendidikan Indonesia Bandung, Universitas Trunojoyo Madura, Universitas Negeri Papua, Universitas Jambi, Universitas Halouleo Kendari, Universitas Sriwijaya, Universitas Ahmad Dahlan Yogyakarta, Universitas Parahiyangan Katolik Bandung, Universitas Yarsi Indonesia, Poltek Negeri Medan, Universitas Islam Indonesia Yogyakarta, University of Malaysia, Politeknik Lampung, Universitas Lampung, Institut Teknologi 10 November Surabaya, Universitas Syarif Hidayat Jakarta, Universitas Maranata Bandung, Universitas Atma Jaya Yogyakarta, Universitas Malahayati. Which reflect the importance of the Internasional Conference on Quantitative Methods Used In Economics And Business.

I hope that this conference will become a place for scientists and economist to share their knowledge and experience and also to promote their expertise in their fields.

This kind of conference will surely have a positive impact on higher education in general as well as development of science, economics, business and research, in particular. For higher education in Indonesia, it is expected that this conference will encourage the faculty members as researchers to do more research as one of their duties.

On behalf of Steering Committee, we would like to express our deepest gratitude to the Foundation Alih Technology, Rector Universitas Malahayati, International Advisory Board members, and also to all participants. We are also grateful to all organizing committee and all the reviewers, without whose efforts such a high standard for the conference could not have been attained. We would like to express our deepest gratitude to the Faculty of Economic Universitasersitas Malahayati for conducted such conference. This is the first International Conference for the Faculty and we expect that this is will become annual activity for the Faculty of Economic.

Bandar Lampung, 15 October 2008

Iing Lukman, Ph.D
The Organizing Chairman

UNIVERSITAS MALAHAYATI
INTERNATIONAL CONFERENCE ON QUANTITATIVE METHODS
USED IN ECONOMICS AND BUSSINESS

PROCEEDING

INTERNATIONAL CONFERENCE QUANTITATIVE METHODS USED IN ECONOMICS AND BUSINESS 2008

on Oktober, 15 – 17th, 2008

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BETA HAT MODEL AND ITS APPLICATION

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ABSTRACT

In linear model $Y = X\beta + \varepsilon$, to test the specific form of a hypotheses of a linear function of parameters β, there are several methods of testing hypotheses available, among others are: Principle Conditional Error, Likelihood ratio test and Invariance test. In this paper, another methods which is well known as Beta hat models (Milliken and Johnson, 2002) will be presented and its application in some cases of linear model will be discussed.

1. Introduction

In linear model $Y = X\beta + \varepsilon$, to test the specific form of a hypotheses of a linear function of parameters β, there are several methods of testing hypotheses of a set of function of parameters or a linear function of parameters. Some well known methods which are used a lot in the literatures are among others: Generalized Likelihood Ratio (Graybill, 1976; Christensen, 1987; Rao, 1973), Principle Conditional Error (Bose,1949; Milliken,1971; Milliken,1997, 2002, Christensen, 1987) and Beta hat model. In a Generalized Likelihood Ratio test, the test

$$\lambda(y) = \frac{\max_{(\beta,\ \sigma^2) in\ \omega} \left(L\ (\beta,\sigma^2,y)\right)}{\max_{(\beta,\ \sigma^2) in\ \Omega} \left(L\ (\beta,\sigma^2,y)\right)}$$

Where the parameter space □□ and □□ are given by

$$\Omega = \{(\beta,\sigma^2)\colon\ \beta\varepsilon\ E_p,\ \sigma^2 > 0\}$$

$$\omega = \{(\beta,\sigma^2)\colon\ \beta\varepsilon\ E_p,\ H\beta = h, \sigma^2 > 0\}$$

The denominator is obtained in a straightforward; it is the likelihood function evaluated at the maximum likelihood value of the parameters β and σ^2. The numerator can be found by two methods: (1) by solving Hβ=h for β, substituting the constraints into the likelihood function, and then maximizing the resulting function; (2) by using Lagrange multipliers and miximizing the likelihood function subject to the constraints Hβ=h.

In Principle Conditional Error, we compare the two Sum of Squares Error from the reduce model, the model which is placing the restriction upon the parameters of another model and Sum of Squares Error from the full model. The test statistic is given by

$$F = \frac{SS(Ho)/df(Ho)}{SSE(R)/df(F)}$$

Where *SSE(F)* denotes Sum of Squares Error after fitting the full model, *SSE(R)* denotes Sum of Squares Error after fitting the reduce model. The Sum of Squares due to the restrictions given by the hypothesis is *SS(Ho)=SSE(R) – SSE(F).* The degrees of freedom for both *SSE(R)* and *SSE(F)* are given by the difference between the number of observations made and the number of parameters estimated. The degree of freedom for *SSE(R)* is denoted by *df(R)* and for *SSE(F)* is denoted by *df(F).* The degree of freedom for *SS(Ho)* is denoted by *df(Ho)* and is calculated from *df(Ho) = df(R) - df(F).*

Beta hat model is built from several models which are have the same form of model to described data from several populations, treatments, or treatments combinations. The beta hat model basically are used the information contained in the parameters and its variance or covariance, and based on these information, then we build the beta hat model. Then the test of hypothesis conducted based on this beta hat model (Milliken and Johnson, 2002).

2. Beta Hat Model

Consider the linear model $Y = X\beta + \varepsilon$ where $\varepsilon \sim N(0,\sigma^2 I)$, then the beta hat model is defined by

$$\hat{\beta} = \mathrm{I}\,\beta + \varepsilon^* \quad \text{where } \varepsilon^* \sim N(0, \sigma^2(X'X)^{-1}).$$

The beta hat model can be used in very general context to give simple understanding, we describe here through example in analysis of covariance model for the jth observation from the ith treatment

$$Y_{ij} = \mu_i + \beta_{1i}X_{1ij} + \beta_{2i}X_{2ij} + \varepsilon_{ij}$$

Or

$$Y_i = X_i\beta_i + \varepsilon_i$$

Where

$$Y_i = \begin{bmatrix} Y_{i1} \\ Y_{i2} \\ \vdots \\ Y_{in_i} \end{bmatrix}, \quad X_i = \begin{bmatrix} 1 & X_{1i1} & X_{2i1} \\ 1 & X_{1i2} & X_{2i2} \\ \vdots & \vdots & \vdots \\ 1 & X_{1in_i} & X_{2in_i} \end{bmatrix}, \quad \beta_i = \begin{bmatrix} \mu_i \\ \beta_{1i} \\ \beta_{2i} \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{in_i} \end{bmatrix}.$$

Least squares of β_i is

$$\hat{\beta}_i = (X'_iX_i)^{-1} X'_iY_i$$

i = 1,2, … , p, and the sampling distribution of $\hat{\beta}_i$ are

$$\hat{\beta}_i \sim N(\beta_i, \sigma^2 (X'_iX_i)^{-1})$$

The SSRes for the ith treatment is SSRes(i) and the pooled estimate of the variance is

$$\hat{\sigma}^2 = SSRes/(N - 3t)$$

Now suppose that we are interested in comparing the β_{li} where l =1,2, ... ,p. parameter from the model, the sampling distribution

$$\hat{\beta}_{li} \sim N(\beta_{li}, \sigma^2 \Gamma_{ikk})$$

Where Γ_{ikk} is the kth diagonal element of $(X'_i X_i)^{-1}$. Let

$$\hat{\beta}'_k = (\hat{\beta}_{k1}, \hat{\beta}_{k2}, \ldots, \hat{\beta}_{kp})$$

And the beta hat model is

$$\hat{\beta}_k = I_p \beta_k + \varepsilon^*$$

Theorem 1

The sum of squares (SS) of testing hypotheses

Ho: $H\beta = h$ against Ho: $H\beta \neq h$

Computaion via the Principle of Conditional Error or likelihood ratio test is equivalent for

$$\hat{\beta} = I \beta + \varepsilon^*$$

for the model

$$Y = X\beta + \varepsilon$$

Proof

Compute SSHo from $\hat{\beta}$- model using Principle Conditional Error (PCE). To test the hypotheses Ho: $H\beta = h$

$\hat{\beta}$- model $\hat{\beta} = I \beta + \varepsilon^*$

$$= I[H^- H + (I - H^- H)]\beta + \varepsilon^*$$

$$= H^- H \beta + (I - H^- H)\beta + \varepsilon^*$$

Impose condition of Ho, replaced $H\beta$ by h. Model under condition of Ho is

$\hat{\beta} - H^- h = (I - H^- H)\beta + \varepsilon^*$ with $\varepsilon^* \sim N(0, \sigma^2 (X'X)^{-1})$

$$\text{JKRes (Ho)} = (\hat{\beta} - H^- h)'(X'X - X'X(I - H^- H))[(I - H^- H)'X'X(I - H^- H)]^- (I - H^- H)X'X)(\hat{\beta} - H^- h).$$

$$= (X\hat{\beta} - XH^- h)'(I - X(I - H^- H))[(I - H^- H)'X'X(I - H^- H)]^- (I - H^- H)X')(X\hat{\beta} - XH^- h).$$

Note that: $(X\hat{\beta} - XH^- h) = (XX^- y - XH^- h)$

$$= XX^- y - XX^- XH^- h$$

$$= XX^-(y - XH^- h)$$

$$\text{SSReg(Ho)} = (y - XH^- h)' XX^-(I - X(I - H^- H)[X(I - H^- H)]^-)XX^-(y - XH^- h)$$

$$= (y - XH^- h)'(XX^- - X(I - H^- H)[X(I - H^- H)]^-)XX^-(y - XH^- h)$$

And

$$F_c = \frac{SSRes(Ho)/q}{\hat{\sigma}^2}$$

Theorem 2
For the linear model

$$y = [X_1 \quad X_2]\begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix} + \varepsilon$$

where y is nx1vector , $[X_1 \quad X_2]$ is nxp matrix, $\begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix}$ is px1 vector of parameter and $\varepsilon \sim N(0, \sigma^2 I)$, the Likelihood Ratio Test statistics for testing the hypotheses

Ho: $H\beta_1 = h$ against Ha: $H\beta_1 \neq h$

Can be obtained from any of the following three models:

(1) $Y = X_1\beta_1 + X_2\beta_2 + \varepsilon$

(2) $\hat{\beta} = \begin{pmatrix}\hat{\beta}_1\\ \hat{\beta}_2\end{pmatrix} = I\beta + \varepsilon^*$, where $\varepsilon^* \sim N(0, \sigma^2 \begin{pmatrix}X'_1X_1 & X'_1X'_2\\ X'_2X_1 & X'_2X_2\end{pmatrix}^{-1})$

Or

(3) $\hat{\beta}_1 = \beta_1 + \varepsilon_1^*$, where $\varepsilon_1^* \sim N(0, \sigma^2 \Delta_{11})$.

Proof

It is already shown that (1) implies (2), now we are going to show that (2) implies (3).
Without loss of generality, we assume that

$\hat{\beta}_1$ has order qx1 and $\hat{\beta}_2$ has order (p-q)x1

So

$\begin{pmatrix}\hat{\beta}_1\\ \hat{\beta}_2\end{pmatrix}$ has order px1

Let

$$\hat{\beta} = \begin{pmatrix}\hat{\beta}_1\\ \hat{\beta}_2\end{pmatrix} = I\beta + \varepsilon^* \quad \text{and } \varepsilon^* \sim N\left(0, \ \sigma^2 \begin{pmatrix}X'_1X_1 & X'_1X_2\\ X'_1X_2 & X'_2X_2\end{pmatrix}^{-1}\right).$$

$\hat{\beta}$ is linear function of multivariate normal, so $\hat{\beta}$ is normally distributed with

$$E(\hat{\beta}) = \beta \quad \text{and} \quad Var(\hat{\beta}) = \sigma^2 \begin{pmatrix}X'_1X_1 & X'_1X_2\\ X'_1X_2 & X'_2X_2\end{pmatrix}^{-1}.$$

Now use Theorem 3.3.5 (Graybill, 1976) and let a matrix $A = (I_q \quad 0)_{qxp}$. And let that $\Gamma = A\hat{\beta}$

Then by Theorem 3.3.5 (Graybill, 1976) Γ is normally distributed with mean

$$E(\Gamma) = E(A\hat{\beta})$$
$$= (I_q \quad 0)_{qXp} \begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix}_{px1}$$
$$= \beta_1.$$

And variance

$$Var(\Gamma) = A(Var(\hat{\beta}))A'.$$
$$= (I_q \quad 0)_{qxp}\ \sigma^2 \begin{pmatrix}X'_1X_1 & X'_1X_2\\ X'_1X_2 & X'_2X_2\end{pmatrix}^{-1} \begin{pmatrix}I_q\\ 0\end{pmatrix}_{pxq}$$

By Theorem 1.3.1 (Graybill, 1976) it can be shown that, the variance is

$$Var(\Gamma) = \sigma^2(X'_1X_1 - X'_1X_2(X'_2X_2)^{-1}X'_2X_1)^{-1} = \sigma^2\Delta_{11}.$$

Then the beta hat model,

$$\Gamma = A\hat{\beta} = \hat{\beta}_1 = \beta_1 + \varepsilon_1^*, \quad \text{where } \varepsilon_1^* \sim N(0, \sigma^2\Delta_{11}).$$

Now to test the hypothesis

Ho: $H\beta_1 = h$ against Ha: $H\beta_1 \neq h$

$$SSHo = \hat{\beta}'_1(\Delta_{11}^{-1} - \Delta_{11}^{-1}H(H'\Delta_{11}^{-1}H)^{-1}H\Delta_{11}^{-1})\hat{\beta}_1$$

The test statistic

$$F_c = \frac{SSHo/r}{\hat{\sigma}^2}$$

Which is distributed as a nocentral *F* with *r* and N-t(q+1) degrees of freedom where *r* is the number of linearly independent parameters in $H\beta_1$ or the rank of *H*.

Example of Application of Beta Hat Model
In this example we will discuss the data from Graybill (1976, p 337). Assume that the data come from three models

$$Y_{ij} = \alpha_i + \beta_i x_{ij} + \varepsilon_{ij} \text{ where } \varepsilon_{ij} \sim N(0, \sigma^2)$$

The hypothesis of interest is

$$Ho = \beta_1 = \beta_2 = \beta_3 \text{ vs Ha: not Ho.}$$

The data are given below:

Y_{1j} X_{1j}	1.2 1.8 1.9 2.1 2.8 2.8 3.1 4.2 4.5 6.2 2.94 1.43 0.48 1.42 -0.96 -1.20 0.01 -1.85 -3.26 -6.73
Y_{2j} X_{2j}	1.0 1.8 2.3 2.5 2.6 3.1 3.4 3.6 3.8 4.2 5.3 6.3 6.5 7.1 7.3 7.6 4.39 5.49 4.03 0.50 2.04 -1.72 -0.04 2.75 -1.52 -2.58 -4.98 -4.53 -7.75 -6.60 -7.15 -8.17
Y_{3j} X_{3j}	-2.0 -1.8 -0.6 0.4 0.5 0.6 1.2 1.5 1.9 2.0 3.8 5.2 9.61 7.78 6.75 4.39 4.05 1.93 3.88 3.65 2.49 3.71 -1.50 -1.29

Sources: Graybill (1976, p 337).

By analysis covariance approach and using SAS program

```
Proc GLM;
Class trt;
Model Y = trt  x  x*trt;
```

From the test statistic we have p-value=0.0847 > 0.05. So we conclude that we fail to reject Ho, ie. The slopes are not significantly different. See the printout below:

SOURCE	DF	SS	F VALUE	P-VALUE
TRT	2	4.7460	5.44	0.0092
X	1	93.9526	215.45	0.0001
X*TRT	2	2.3278	2.67	0.0847

By beta hat model, the model

$$\hat{\beta} = I\,\beta + \varepsilon^*$$

From the computer print out, the estimation of β and its standard error (SE),

$$\hat{\beta}' = (-0.5326 \quad -0.4388 \quad -0.5982)$$

$$SE = \begin{pmatrix} 0.0469 \\ 0.0405 \\ 0.0683 \end{pmatrix}.$$

By using the estimate of β ($\hat{\beta}$) and corresponding standard error (SE) one can use the following SAS Program to find SSHo.

```
DATA BETAH;
INPUT TRT  BHAT  SE;
S2=0.6604;
V=SE/S2;
WT=1/(V*V);
```

```
CARDS;
1      -0.5326  0.0469
2      -0.4388        0.0405
3      -0.5982        0.0683;
PROC REG;
MODEL  BHAT= ;  WEIGHT   WT;
```

The output is

TEST FOR EQUAL BETA

OBS	TRT	BHAT	SE	S2	V	WT
1	1	-0.5326	0.0469	0.6604	0.07102	198.275
2	2	-0.4388	0.0405	0.6604	0.06133	265.891
3	3	-0.5982	0.0683	0.6604	0.10342	93.492

Source	df	SS	MS	Fvalue	ProbF
Error	2	2.35304	1.17652	.	.

The error sum of squares provides the sum of squares due to deviations from the null hypothesis that all of the parameters are equal. The resulting F statistic is F=1.17652/0.43615 = 2.697 and from the F-table $F_{0.05,\,2,32}$ = 3.29 and we fail to reject Ho.

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