



## A STUDY ON REACTIVE POWER ALLOCATION FOR ELECTRICAL POWER DISTRIBUTION SYSTEM WITH LOW VOLTAGE PROFILE

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### ABSTRACT

Due to its long feeder line with high R/X ratio, a distribution system suffers from low voltage profile at its load nodes. This work studies reactive power allocation for this type of problem. A linear programming-based optimal power flow is proposed to solve this problem by finding minimal amount of installation of new reactive power support devices while maintaining voltage at each load bus within the  $\pm 5\%$  deviation. The proposed approach was tested on 11-bus test system and a real-world distribution feeder in Indonesia with 119 buses and 106 load points. This actual feeder experiences voltage magnitude below the 0.95 p.u limit. Simulation results show that some load points of this feeder require installation of new reactive power support devices to maintain their voltage to be above the 0.95 p.u. limit.

**Keywords:** reactive power allocation, distribution system, low voltage profile, linear programming.

### INTRODUCTION

A Distribution system under heavily loaded condition with a very long feeder usually experiences low voltage profile. A very long distribution feeder is common in rural area with sparse loads. Clearly, this is not an option, however, slow transmission system expansion has forced the distribution company to face this situation. Such situation requires the electric distribution company to take action to improve the voltage profile. Several actions are available such as installing voltage regulator, installing distributed generator, feeder reconfiguration, or installing reactive power support.

Previous works on dealing with reactive power allocation for the distribution system have been conducted. For example, works in [1]-[6] proposed the use of artificial intelligence-based method to solve the reactive power allocation in distribution networks. Installing capacitor has been also an option for reducing losses as well as improving harmonics performance of a distribution system [7]-[9]. It is clear from these previous works that reactive power plays an important role in the operation of distribution networks.

This work proposes a formulation to improve the low voltage profile in a distribution system by means of linear programming with rectangular voltage coordinates. This is an extension to the previous work on Newton-Rapshon power flow in rectangular coordinates [10] and the linear programming optimal power flow based on polar voltage coordinates [11]. The proposed method is then applied to solve reactive power allocation problem by means of capacitor installation in a distribution system to improve its voltage profile.

### PROBLEM FORMULATION

A long distribution feeder together with heavy and sparse load usually suffers from low voltage profile. As it is well-aware in AC electrical power that voltage magnitude is closely related to reactive power flow in the network, changing reactive power flow should improve

voltage profile. However, in a system with a very low voltage profile, solving the power flow problem and simulating various solution scenarios with the conventional power flow tool may be difficult since the Jacobian of Newton-Rapshon power flow is singular around the point of voltage collapse. This phenomenon has been studied in power systems research for long. This has been summarized in [12] that there are two approaches for solving this type of problem i.e. continuation power flow and optimal power flow.

In order to overcome this, a linear programming-based optimal power flow is adopted and a linear slack variable is introduced to the problem. This slack variable indicates how the distribution system operator should take a necessary action to bring the voltage to the safe level. As already mentioned above about the relation between voltage and reactive power, the introduced slack variable is modeled to accommodate reactive power injection by additional VAR support device in the distribution system. This injection is different from active and reactive power injections in the substation. These slack variables during the iterative process are able to move in the positive and negative directions to bring the solution to the feasible region. Since there is a cost associated with installing a new VAR support device, the problem is solved by minimizing the cost of new VAR device installation. Detail of the formulation is provided in the next section.

### REACTIVE POWER ALLOCATION MODEL

Application of linear programming to optimal reactive power allocation in the transmission system has been previously developed in [11]. The difference in formulation with this current work is in the use of full rectangular coordinates while the formers utilized voltage in phasor coordinates. Consequently, in this current work, no trigonometric function is evaluated and the formulation power flow equations are different from the previous works. Minimizing cost of installing new VAR source is



taken as the objective function. Below is the optimization model in the proposed method.

$$\text{Min. } \sum_{n \in \{\text{load buses}\}} C_n^+ Q v_n^+ \tag{1}$$

Subject to:

$$\frac{\partial P_i}{\partial e_i} \Delta e_i + \frac{\partial P_i}{\partial f_i} \Delta f_i + \frac{\partial P_i}{\partial e_k} \Delta e_k + \frac{\partial P_i}{\partial f_k} \Delta f_k - P_m^+ + P_m^- = -P_i + P_i^0 \tag{2}$$

$$\frac{\partial Q_i}{\partial e_i} \Delta e_i + \frac{\partial Q_i}{\partial f_i} \Delta f_i + \frac{\partial Q_i}{\partial e_k} \Delta e_k + \frac{\partial Q_i}{\partial f_k} \Delta f_k - Q_m^+ + Q_m^- - Q v_n^+ + Q v_n^- = -Q_i + Q_i^0 \tag{3}$$

Bounds are defined as:

$$\forall i \in \{\text{all buses}\}: \begin{aligned} \Delta e_{min} &\leq \Delta e_i \leq \Delta e_{max} \\ \Delta f_{min} &\leq \Delta f_i \leq \Delta f_{max} \end{aligned} \tag{4}$$

$$\forall m \in \{\text{substation bus}\}: \begin{aligned} P_{min}^+ &\leq P_m^+ \leq P_{max}^+ \\ Q_{min}^+ &\leq Q_m^+ \leq Q_{max}^+ \\ P_{min}^- &\leq P_m^- \leq P_{max}^- \\ Q_{min}^- &\leq Q_m^- \leq Q_{max}^- \end{aligned} \tag{5}$$

$$\forall n \in \{\text{load buses}\}: \begin{aligned} Q v_{min}^+ &\leq Q v_n^+ \leq Q v_{max}^+ \\ Q v_{min}^- &\leq Q v_n^- \leq Q v_{max}^- \end{aligned} \tag{6}$$

Where

$C_n^+$  is cost coefficient of slack variable for adding reactive power support device in positive direction.

$C_n^-$  is cost coefficient of slack variable for adding reactive power support device in negative direction.

$Q v_n^+$  is slack variable for adding reactive power support device in positive direction.

$Q v_n^-$  is slack variable for adding reactive power support device in negative direction.

$P_m^+$  is slack variable for adding active power injection in positive direction for substation bus.

$P_m^-$  is slack variable for adding active power injection in negative direction for substation bus.

$Q_m^+$  is slack variable for adding reactive power injection in positive direction for substation bus.

$Q_m^-$  is slack variable for adding reactive power injection in negative direction for substation bus.

$P_i^0$  is scheduled active power injection for all buses

$Q_i^0$  is scheduled active power injection for all buses

Rectangular representations of voltage and power are as follow:

$$\dot{V}_i = e_i + j f_i \tag{7}$$

$$P_i = e_i \sum_{\substack{k=i \\ k \in i}} (G_{ik} e_k - B_{ik} f_k) + f_i \sum_{\substack{k=i \\ k \in i}} (G_{ik} f_k + B_{ik} e_k) \tag{8}$$

$$Q_i = f_i \sum_{\substack{k=i \\ k \in i}} (G_{ik} e_k - B_{ik} f_k) - e_i \sum_{\substack{k=i \\ k \in i}} (G_{ik} f_k + B_{ik} e_k) \tag{9}$$

$\dot{V}_i$  = complex voltage at bus  $i$

$e_i, e_k$  = real part of complex voltage at bus  $i$  and  $k$

$f_i, f_k$  = imaginary part of complex voltage at bus  $i$  and  $k$

$P_i$  = active power injection at bus  $i$

$Q_i$  = reactive power injection at bus  $i$

$G_{ik}, B_{ik}$  = real and imaginary part of Bus Admittance Matrix (**Y-bus Matrix**) with respect to the admittance from bus  $i$  to bus  $k$

### RESULTS AND DISCUSSIONS

In order to demonstrate the effectiveness of the proposed approach to solve reactive power allocation in distribution system, a simplified feeder derived from an actual distribution feeder is considered. This test system comprises eleven nodes as in Figure-1.

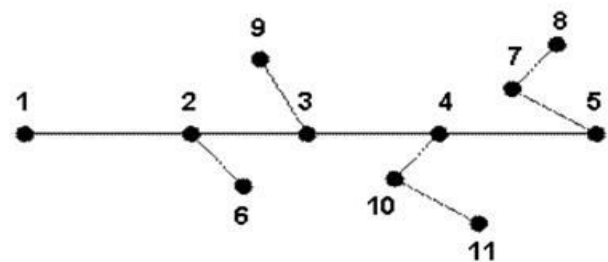


Figure-1. An 11-bus radial test system.

A base scenario of the above system is simulated for comparison of the proposed method and the conventional Newton-Raphson power flow. In order to



allow the proposed algorithm to provide base case power flow, all voltages are relaxed. Results are given in Table-1. The differences observed between the two methods are minimal and negligible.

**Table-1.** Base scenario voltage comparison.

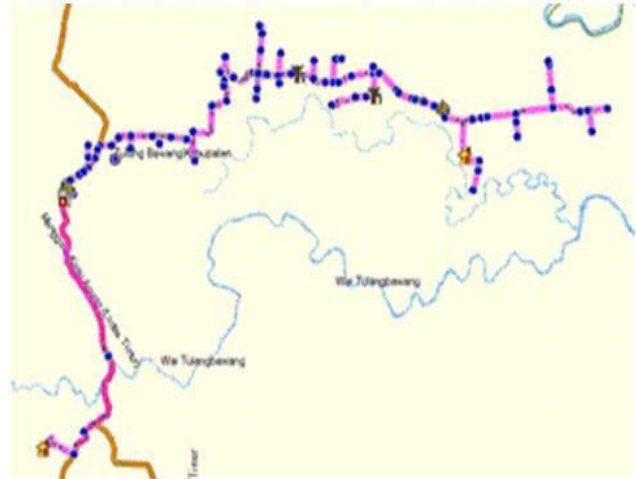
Bus #	Voltage Magnitude	
	Conventional N-R	Proposed Method
1	1.0000	1.0000
2	0.7945	0.7945
3	0.7591	0.7591
4	0.7505	0.7505
5	0.7484	0.7484
6	0.7939	0.7939
7	0.7477	0.7477
8	0.7472	0.7472
9	0.7537	0.7537
10	0.7362	0.7362
11	0.7312	0.7312

The results of base scenario are satisfactory and show that the proposed method is working. Next stage is to apply the proposed method to search for the required amount of reactive power such that all bus voltages are within the specified limits. As can be seen in Table-1, all nodes except bus 1, which is the main substation bus, suffers from voltages below the operational constraint of 0.95 p.u. Improving the voltage profile in this work can be achieved by installing reactive power device. Assumption was made by setting the cost of installing the new VAR support device to be flat for all nodes. From the base scenario results, except the main substation bus, all nodes experience low voltage profile, below the 0.95 p.u., and it can be expected that the new VAR support device should be installed there.

**Table-2.** Improvements in voltage profile and required VAR support device.

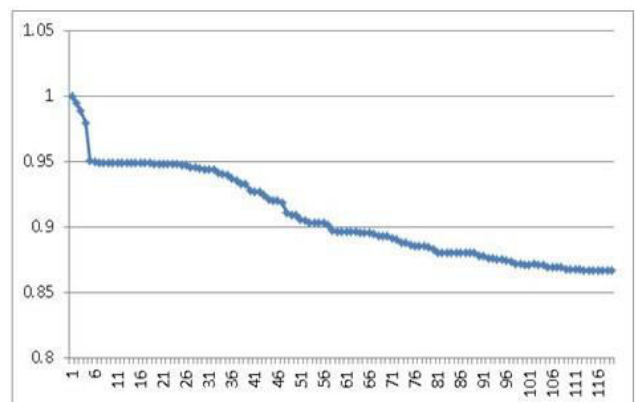
Bus #	Base Scenario Voltage Magnitude	Improved Voltage Magnitude	New VAR Device Installation
1	1.0000	1.0000	-
2	0.7947	0.9680	-
3	0.7594	0.9523	-
4	0.7508	0.9523	-
5	0.7487	0.9509	-
6	0.7942	0.9675	-
7	0.7480	0.9505	-
8	0.7475	0.9502	0.097
9	0.7540	0.9503	1.119
10	0.7365	0.9501	1.326
11	0.7315	0.9501	2.124

The required reactive power installation is suggested at four buses, i.e. Buses 8, 9, 10, and 11. The amount of additional reactive power support also shows the distance of the base case from the required operating conditions as stated in the National Grid.



**Figure-2.** GPS coordinates plot of load points (blue dots) from an actual distribution feeder.

The proposed method was also applied to a real-world distribution feeder with low voltage profile in Lampung Province. The feeder is assumed to be balanced three-phase and therefore modeled in this work as a single-phase network. Base case simulation of this feeder shows that most of its load nodes are operated at below the 0.95 p.u. limit. This feeder is typically found at rural area supply and characterized with high R/X ratio and sparse loads. Figure-2 shows the coordinates plot of an actual feeder. Main substation is situated at the left-bottom of the picture and there exists only three load points spanned along the first approximately 30 km from the main substation.



**Figure-3.** Voltage profile of 119-bus actual 20 kV feeder for the base scenario.

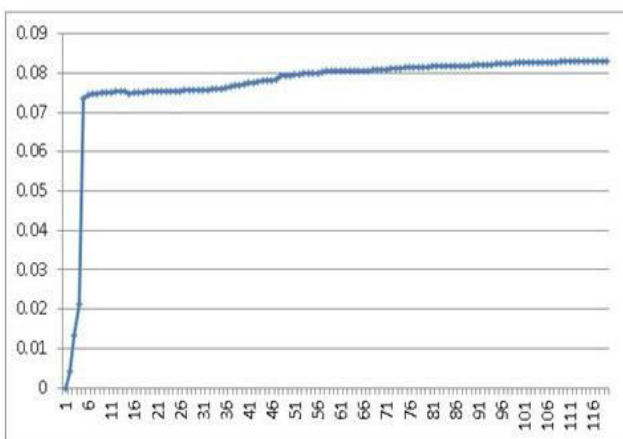
From Figure-3, it can be seen that voltage drops to approximately 0.86 p.u. at the furthest load bus. In order



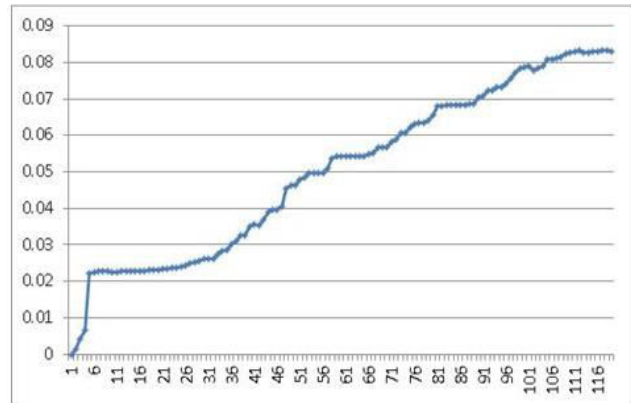
to improve this voltage profile, installation of new reactive power device to support voltage at the distant nodes is required. The authors investigated two compensation strategies i.e. (1) concentrated compensation near the main substation and (2) distributed compensation along the feeder. In order to simulate these strategies, cost coefficients in the objective function are modeled differently. The first compensation strategy is approached by setting cost coefficient of new VAr installation to be higher as the distance from the main substation increases. Distributed compensation is achieved by setting flat cost coefficient for all candidate nodes. This latter assumption is acceptable if the actual cost of installation does not vary significantly within 100 km of distance which is the case of the studied feeder of Figure-2.

The first compensation strategy will bring the solution to the nodes nearer to main substation. For a very long feeder line with such large voltage drop at end nodes, this may lead to a new problem of overcompensated system and may further increase system losses. The latter assumption is expected to provide solution just about the location that requires compensation. Another important consideration taken into simulation was the reactive power capacity of capacitor bank. The distribution company of this area preferred the use of 200 kVAr capacitor bank. Therefore, in our formulation, we accommodate the 200 kVAr capacitor as the maximum new reactive power injection.

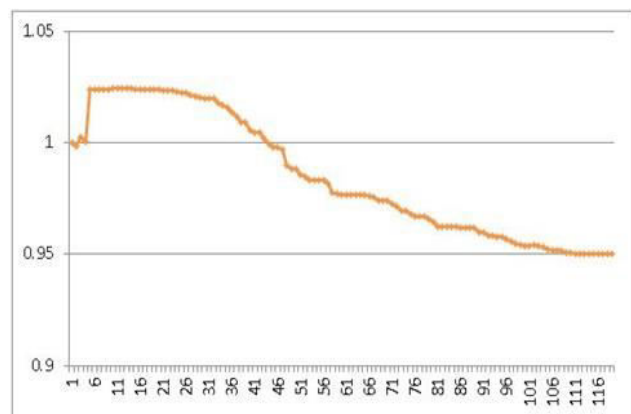
Figure-4 and Figure-5 show voltage magnitude improvement for the first compensation strategy and the second compensation strategy respectively. The second strategy provides better solution than the first scheme in terms of improvement at nodes with lower voltage. As it is validated in Figure-5, voltage improvement increases along the feeder and highest improvement is at the furthest nodes. On the other, the first strategy brings voltage improvement to those nodes near to main substation which previously do not suffer from poor voltage profile.



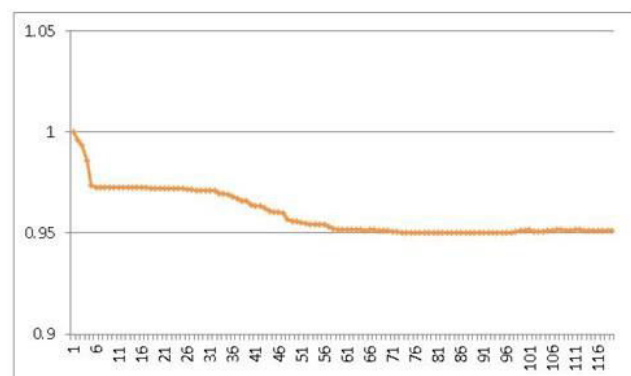
**Figure-4.** Voltage improvement of first compensation strategy.



**Figure-5.** Voltage improvement of second compensation strategy.



**Figure-6.** Voltage profile for first compensation strategy.



**Figure-7.** Voltage profile for second compensation strategy.

Figure-6 and Figure-7 compare voltage profile of the studied distribution system with two compensation strategy. Compensating reactive power for voltage at the requiring nodes as in the second strategy provides better solution i.e. voltage variations of the overall system are within the range of 5%. If the first strategy is taken as the consideration for determining reactive power allocation of this kind of system, then all capacitors will be placed at nodes nearest to the main substation. In our simulation, 17 capacitors of 200 kVAr are required and installed from bus



5 to bus 21. This results in large increase of voltage at the first nodes to compensate drop at the further nodes as shown in Figure-6. Consequently, the system becomes over compensated and the main substation is operated in leading power factor as large amount of reactive power flows back to the main substation. This compensation also increases system active power losses to about 80% from the base-case. This is not the case with the second strategy, as the required reactive power compensations are more evenly distributed along the feeder. The latter compensation reduces active power losses to about 13% from the base-case. Total reactive power installed is about 1 MVar which is much less than the first cost scheme i.e. 3.4 MVar. Clearly, distributed compensation of the required reactive power has better contribution to both improving voltage profile and reducing system active power losses.

## CONCLUSIONS

This work studied reactive power allocation problem in distribution feeder with low voltage profile. A linear programming model was developed to investigate two compensating strategies i.e. concentrated and distributed compensation. After simulating for the actual 20 kV distribution feeder, the following conclusions are drawn:

- Distributed compensation strategy provides better voltage improvement as well as active power loss reduction. In the studied distribution system, about 13% of loss reduction was achieved by utilizing the distributed compensation strategy while the concentrated compensation strategy increased active power loss to 80% from the base scenario.
- Voltage variation is within 5% when reactive power is allocated using the second compensation strategy, while the first compensation strategy holds for about 8% voltage variation.
- The concentrated compensation strategy leads the system to over compensated situation. This means large amount of reactive power is injected to the system which is much more than the required one to improve the voltage. This study shows that concentrated compensation strategy suggests about 3.4 MVar of reactive power while the second strategy only requires about 1 MVar of reactive power for the 119-bus actual 20 kV distribution feeder.

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