Journal of Physics: Conference Series

PAPER • OPEN ACCESS

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To cite this article: Rini Asnawati and Caswita 2018 *J. Phys.: Conf. Ser.* **1028** 012121

IOP Conf. Series: Journal of Physics: Conf. Series **1028** (2018) 012121 doi:10.1088/1742-6596/1028/1/012121

Redefinition of the Kinds of Quadrilateral Based on the Angles and Sides

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Abstract. Geometry is a branch in mathematics having important and strategic role to develop learners' spatial and higher order thinking skills. Both of them can be achieved by learner whenever the concepts of geometry are studied through inqury-discovery process. Therefore, redefinition of all kinds of convex qaudrilateral and their properties through analysis and investigation with respect to measure of its angles or sides. This study are done to have primely structure of convex quadrilateral that implies to acheive the objective of geometry learning. The method of study is initiated by reviewing and evaluating recent references discussing definition and property of quadrilateral, and then create what should be invented. Based on the study has been done, it was obtained many kinds of quadrilateral based on angles or/and sides and then their property that shows primely structure of convex quadrilateral concepts.

1. Introduction

The Geometry is a part of mathemthics that is an important and strategic thing to develop learners' spatial and higher order skills. Regarding to this, in [1] mentioned that the objective of geometry learning related to the spatial sense and geometric reasoning. The spatial sense means an intuitive feel for shape of geometry object that includes an ability to recognize, visualize, represent, and transform geometric shapes. Whereas geometric reasoning means reasoning by using critical thinking, logical arguments, and spatial thought to solve problem and find relationships. Both skills should be possessedby learners after learning. Therefore, as anintegral part of mathematics, in [2] mentioned that geometry education must satisfy six principles, that is equity, curriculum, teaching, learning, assesment, and technology. All of six principles are expected support and facilitate students to developtheir ability involving problem solving, reasoning and proof, communication, connection, and representation. The another idea of geometrical thinking processproposed by Pierre van Hiele and Dina van Hiele-Geldof that has been discussedin [3-5]. The van Hiele levels of geometric thought consists offive, that visualization, analysis, informal deduction, deduction, and rigor. These levels describes how learner thinks and what types of geometric ideas constructed and developed, rather than how much knownledge grasp.

In the last 10 years, learner's mathematical thinking skills, included gemetrical thinking,are still stagnant in below level. It is shown the result of survey done by TIMSS (Trends in International Mathematics and Science Study) in the year 2011 and 2015 in [6-7] that the average or learner's mathematical thinking skill from many participant countries are below 500, especialy in reasoning and problem solving needed spatial sense and geometric reasoning. The relative similar data is shown from the result of survey done by PISA (Programme for International Students Assessment) in the years

2009, 2012, and 2015 in [8-10] that the average of mathematical thinking score of participant from OECD (the Organisation for Economic Co-Operation and Development) countries are respectively 496, 494, and 490, while the average score of participants not from OECD country is more lower.These conditions is caused by many factors, one of them is fallacy in develoving didactical design.

Didactical design has an strategic role in improvement the quality of mathematics teaching and learning. Therefore, the didactical design is veryimportant and interesting in order to improvement teaching and learning process. Therefore, many researchrs are interested to the study of didactical design in the mathematics learning as mentioned in [11]. In [12] discussed about various terms and concept spread in the international community that deals with mathematics eduation.

Didactical design discusses about relation in learning process between teacher, students, and subject matters that describedas didactical triangle.According to [13], the relations in learning only involvesrelations between: student-subject matter (well-known as didactical relation) and teacher-student (as well-known pedagogical relation). Then, in [14] states that the relation between teacher-subject matter should be attention. It is an important thing to anticipate response by student in the form of questions or objection to concepts given by teacher or in learning resources. In learning process, it is frequently confronted obstacle, that is learning and didactical obstacle, as well as didactic-pedagogic anticipation. In [15] mentioned that in learning is often confronted with didactic obstacle. The learning obstacle is also frequently happened when learning process is minimal guidence as mentioned in [16]. It is caused that minimal guidence implies with respect less activities by learners so that learning to be meaningless. However, In this paper only focused to handle didactic obstacle through improving structure of quadrilateral concepts and procedures of its construction.

In the study classification of triangle, all references discussmore clearly by considering measure of angles or sides. Indeed, the study of classification of triangle based on angles and sides is usually not discussed or intentionally left for learner. However, when study of classification of quadrilateral is not so clearly to be understood. Many learners and teachers do not know when asked based on what a quadrilateral is so-called rectangle, paralellogram, trapezium, kite, rhombus, and square. All of kinds of those quadrilateral are well-known, but only as collection of meaningless things. This causes that learning in quadrilateral does not give impact to spatial skill and geometrc reasoning.Therefore, redefinition of quadrilateral is a study that important and must be urgently so that learner achieves high level thought of van Hiele.

2. Method of Study

The method of research is begun with study of many references that discussed quadrilateral including definition and property. All of those are reviewed and evaluated to find out definition and property of quadrilateral more meaningful. These activities aim to have the information about established concepts in quadrilateral that probably cause learner is getting difficulty to grasp the concepts of geometry and causing spatial thought undeveloped. Moreover, these activities are also expected to be base in developing didactical design of concepts of quadrilateral, especially didactic design, to facilitate spatial thought and geometric reasoning.

Based on study in many references that quadrilateral is a simple closed curve having four sides formed straight lines. Indeed, besides having four sides the quadrialateral also has four angles so that quadrilateral has other name so-called quadrangle. In references [1,17] properties of quadrilateral related tomeasure of angles or sides stated on the following theorem.

Theorem 1. Let plane ABCD be a quadrilateral, then

(i) The sum of measure of its angles are 360^o or 2π radian, that is:

$$m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360^o \quad (1)$$

(ii) The sum of length of three sides is greater than length of other side:

$$a + b + c > d, \quad (2)$$

where a, b, c and d are length of sides.

In References [1,17], the quadrilateral can be classified to two kinds, that is convex and concave quadrilateral. The convex quadrilateral means that the measure of all angles of quadrilateral are less than 180^o(a segment connecting arbitrary two points inside is inside of quadrlateral), the otherwise it is called concave quadrilateral (a segment connecting two points inside is not inside). As illustration, two kinds of quadrilateral can be seen in figure 1.

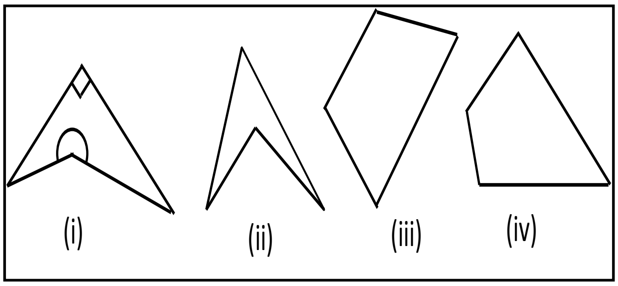

Figure 1. Shapes of Quadrilateral Based on Angles

Figures 1(i) and (ii) are concave quadrilateral, while figures 1(iii) and (iv) are convex quadrilateral. The other shapes of both kinds can be obtained by considering the kinds of angles, for instance "obtuse-concave quadrilateral.It is easy to see that the study of concave quadrilateral based on angles is too simple, because there are just three kinds, that are obtuse-concave quadrilateral, right-concave quadrlateral, and acute-cancave quadrilateral. As well, the study of concave quadrilateral based on sides is relatively simple. Therefore, the scope of study in this paper only discusses convex quadrilateral because the other one is more simple to be studied.

Based on the scope of study and implication of Theorem 1, redefinition of quadrilateral and analysis of their property will be done with referring to van Hiele levels of spatial sense and geometric reasoning.

3. Results

The study is initiated by analysing Theorem 1 to meet the objective of research. The results of study are in the form definitions and theorems about convex quadrilateral discussed as follows.

3.1. The Kinds of Quadrilateral Based on Angles and Their Property

According to its measure, the angle can be classified to be three kinds, that is acute angle, right angle, and obtuse angle. Based on these kinds of angles and the Theorem 1(i), it is obtained the following theorem.

Theorem 2. Let plane ABCD be a convex quadrilateral, then one of the following properties hold:
- (i) all angles of the plane ABCD are right angle;
- (ii) two angles of the plane ABCD are right angles and two other angles are acute and obtuse;
- (iii) one angle of the plane ABCD is right angle; and
- (iv) there is no right angle.

Proof of Theorem 2 is obvious and left for learner in order to has thought at deduction level. It is easy to see that the corollary of Theorem 2(iii) and (iv).

Corollary 3. The following statements hold:
- (i) If the plane ABCD has property (iii) in Theorem 2, then the other angles are: a) two obtuse angles and one acute angle or b) one obtuse angle and two acute angles; and
- (ii) If the plane ABCD has property (iv) in Theorem 2, there are possibilities: a) three obtuse angles and one acute; b) two obtuse angles and two acute angle; c) one obtuse and three acute angles.

The property (i.a) in Corollary 3, there are two possibilities, that is the two obtuse angles are adjacent or opposite and the same case for property (1.b) in Corollary 3.

IOP Conf. Series: Journal of Physics: Conf. Series **1028** (2018) 012121 doi:10.1088/1742-6596/1028/1/012121

Referring to Theorem 2 and Corollary 3, it could be redefined well-known quadrilateral and some unwell-known quadrilateral as follows.

Definition 4: Let plane ABCD be a quadrilateral, then

(i) it is calledrectangle if (and only if) the property (i) in Theorem 2 is satisfied;
(ii) it is called right trapezium if (and only if) the property (ii) in Theorem 2 is satisfied;
(iii) it is called ”right quadrilateral” if (and only if) the property (iii) in Theorem 2 is satisfied;
(iv) it is called parallelogram if the corollary 3(ii)(b) is satisfied and two pair of opposite angles (has no common arm) are equal; and
(v) it is called trapezium if the corollary 3(ii)(b) is satisfied and sum of a pair of adjacent sides (having a common arm) is 180^o (π rad).

The “blank quadrilateral” can be defined by applying the other cases in Corollary 3(i) and (ii), for instance the “acute quadrilateral” (having three obtuse angles and one acute angle).In order that learner has thought at level visualization or recognation and represent of figures types based on angles as mentioned in[1,3], geometric objects in Definition 4are represented on Figure 2. It is an important in order that learner can groupings of shapes based on angles that seen to be alike.

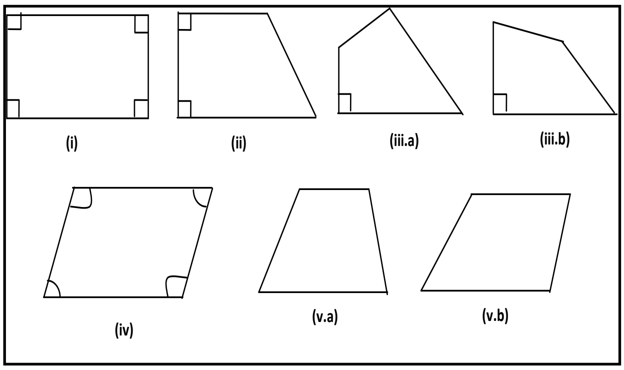

Figure 2. Shapes of Quadrilateral Based on Angles

In figure 2, there are two figures (iii. a and iii.b) showing“right quadrilateral”, another possibility that both obtuse angles are adjacent with respect to right angle; and Figure (v.a) and (v.b) can be considered as different trapezium, because of position of obtuse angle and acute angle, that is they are adjacent angles and opposite angles.

By doing analysis with respect to angles to achieve analysis level of van Hiele [1,3], it follows that rectangle can be called as paralleogram, (right) trapezium, and right quadrilateral; whereas paralellogran can be called trapezium but not right trapezium. The next level though of van Hiele is informal deduction, that is doing informal discovery to find property of quadrilaterals based on angles. These skills can be achieved by learner by giving meaningful experiences when reading,discussing, and understanding subject matters. As for the property of the kinds of quadrilateral in Definition 4 stated in the following theorem.

Theorem 5. The following statements hold.

(i) A rectangle has property: two pairs of opposite sides are same in long; both diagonal are equal and bisection; and four symmetry (two rotation and two folded symmetry to horizontal and vertical lines) that form group to composition operation;
(ii) Right trapezium, right quadrilateral, and trapezium have property: intersection of diagonal inside and one rotating symmetry; and
(iii) Paralellogram has property: intersection point of diagonals inside and bisection, two pairs of opposite sides are equal in long, and two rotation symmetry.

The proof of above theorem is easy so that left it. Referring to Theorem 5(iii) and because two pairs of opposite sides are equal in long imply having two pairs of opposite sides are equal in long and vice versa, then paralellogram can defined based on side, that is quadrilateral having two pairs of opposite sides equal in long (having two pairs of opposite sides parallel) as in many references.

3.2. *The Kinds of Quadrilateral Based on Sides and Their Property*

After discussing the quadrilateral based on angle, it is now studied quadrilateral based on measure of side. Referring to Theorem 1(ii) and the length of side, it is obtained the following theorem.

Theorem 6. Let plane ABCD be a quadrilateral, then one of the following properties holds:
(i) all sides of the plane ABCD are equal;
(ii) three sides of the plane ABCD are equal in long and another side is different in long;
(iii) two sides of the plane ABCD are same in long; and
(iv) all sides are not different in long.

The proof of the above theorem is obvious so that left to learners. It is easy to have the corollary of Theorem 6(iii) that stated in the following corollary.

Corollary7. Based on the Theorem 6(iii), it implies that:
If the plane ABCD has property (iii) in Theorem 6, the other two sides can be:(a) equal to each other or (b) differentsides in long.

The property (a) and (b) in Corollary 7 severally two possibilities, that is the pair equal sides is adjacent and opposite so that it can be considered as different kind of quadrlateral.

Referring to Theorem 6and Corollary 7, it is redefined well-known quadrilateral and some "unwell-known" quadrilateral as follows.

Definition8: Let plane ABCD be a quadrilateral, then
(i) it is called rhombus if (and only if) the property (i) in Theorem 6 is satisfied;
(ii) it is called kite if (and only if) the property (iii)(a) in Corollary 7 is satisfied and two pair of adjacent sides are equal in long;
(iii) it is called "isoscelesquadrilateral" if (and only if) the property (ii) in Theorem 6 is satisfied and both are opposite sides (has no a common end point); and
(iv) it is called "scalene quadrilateral" if the property (iv) in Theorem 6 is satisfied.

The "blank quadrilateral" can be defined by applying the other cases in Theorem 6(ii) and Corollary 7, for example "equi-three-lateral quadrilateral" having equal three sides in long. To have geometric thought at level visualization or recognation and represent of figures types based on sides, geometric objects in definition 8 are represented on figure 3. It aims in order that learner can groupings of shapes based on sides.

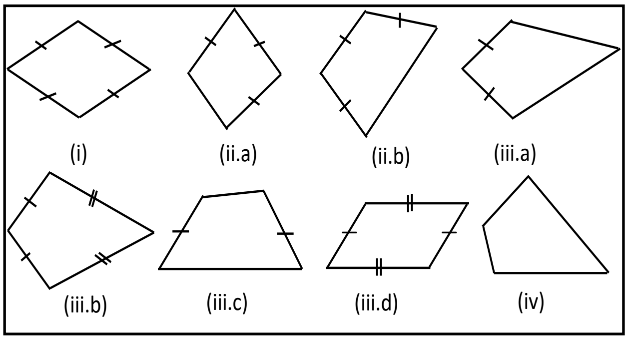

Figure 3. The Shapes of Quadrilateral Based on Sides

The figures 3(i), (iii.b), (iii.c), and (iv) based on the measure of sides are respectively rhombus, kite, isoceles quadrilateral, and scalene quadrilateral. Whereas the Figures 2(ii.a), (ii.b), ((iii.a), and (iii.d) are called "blank quadrilateral", but it is well known that Figure 2(iii.d) based on angles is a paralellogram. Therefore, paralellogram can be defined quadrilateral based on angles or sides.

The level thought of van Hiele is informal deduction and deduction. These level are skills that show that learner can informal discover the properties of among quadrilateral based on sides and as well their relationships. As for the property of the kinds of quadrilateral in Definition 8 stated in the following theorem.

Theorem 9. The following statements hold.
(i) A rhombus has property: two pairs of opposite angles are same; both diagonal are bisection

and perpendicular; and four symmetry (two rotation and two folded symmetry with respect to diagonals) that form group to composition operation;

(ii) Kite has property: intersection of diagonal inside and perpendicular, a pair of opposite angles is equal, as well as two symmetris (rotation and folded); and

(iii) Isoceles and scalene quadrilateral have property: intersection point of diagonals inside and one rotation symmetry.

Because two pairs of opposite angles do not imply two pairs opposite sides are equal, then paralellogram is not a rhombus, but rhombus is a paralellogram. This implies that all property of paralellogram possessed by rhombus, but otherwise does not hold. Proof of above theorem and statements are obvious.

3.3. The Kinds of Quadrilateral Based on Both Angles and Sides.

In the study of triangle, it can be distinguished based on both angles and sides. This implies that there are seven kinds of trangle, that is acute-equilateral triangle, acute-scalene triangle, acute-isoceles triangle, obtuse-isoceles triangle, obtuse-scalene triangle, right-isocele tringle, and right-scalene triangle. This analysis shows ability to deductive axiomatic and rigor for system of geometry. It is an important thing to distinguish what kinds of quadrilateral based on both angle and side. Based on the study referring the the above discussion, it is found the following definition.

Definition 10. Let plane ABCD be a quadrilateral, then

(i) it called square if the properties (i) in Theorem 2 and Theorem 6 are satisfied;

(ii) it is called "isoceles-trapezium" if sum of a pair of adjacent angles is 180^o and a pair of opposite sides is same in long; and

(iii) it is called "right-kite" if it has a right angle and two pairs of adjacent sides are same in long.

Besides the kinds of quadrilateral defined above, many kinds of quadrilateral that are still called"blank quadrilateral". To achieve spatial thought, goemtric objects in Definition 10 are represented on figure 4. It aims in order that learner can distinguish the shapes of quadrilateral based on angles and sides.

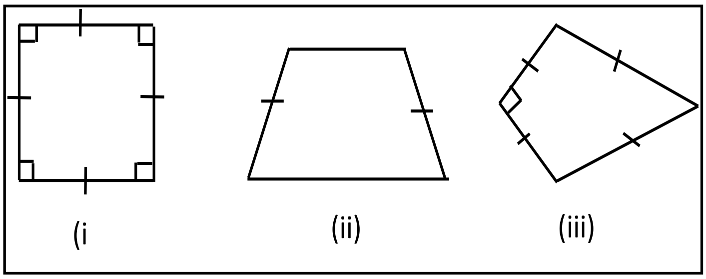

Figure 4. The Shapes of Quadrilateral Based on both Angles and Sides

In fact many kinds of quadrilateral based on both angles and sides can be defined. So, learner can be study quadrilateral more detail. Furthermore, the property of three kinds of quadrilateral in Definition 10 are studied and the result stated on the following theorem.

Theorem11. The following statements hold

(i) The square has properties: the diagonals are same in long, bisection and perpendicular, as well as eight symmetry (four rotation and four folded symmetry) that form group with respect to composition.

(ii) The isoceles-trapezium has property:intersection point of diagonal inside and same in long, two symmetry (one rotation and one folded symmetry (horizontal/ vertical axis).

The right kite has property:intersection point of diagonals inside and perpendicular, and two symmetry (one rotation and one folded symmetry (diagonal).

It is easy to prove theorem above, because all statements in this theorem are just logical consequences, but developing the proof of this theorem would be experience to develop deductive thought and to give ability comparisons and contrasts amoungs different kinds of quadrilateral. The other that the development of blank quadilaterals dan their properties can also give experience to construct the spatial sense. It is an important thing to develop subject of study in school geometry that relatively there is no change. This condition can imply to the achievement of objective of school geometry learning.

4. Conclusion

The study of redefenition of convex quadrilateral is not only an interesting in enrichment subject matter in school geometry, but also an important for developing learner's spatial and higher order thinking skill. It is known both skill are very important for everybody to solve real problems. In this paper, definition of the kinds of convex quadrilateral is constructed by analysing properties of quadrilateral based on the angles and/or sides and as well as their property. By doing reviewing and evaluating many references and doing research, it can be contructed many kinds of convex quadrilateral and their properties.Based on the study has been done, it was obtained many kinds of many quadrilateral based on angles or sides and their property that shows primely structure of quadrilateral concepts that probably can be developing spatial and higher order thinking skills.

Acknowledgmenent
The authors would like to thank to the Directorate General of Research and Community Service for financial support through applied product research grantin the year 2017 and University of Lampung for giving us facility and opportunity to do this research well.

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